Thin-Walled Structures 46 (2008) 1183–1191

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Buckling of laminated sandwich plates with soft core based on an improved higher order zigzag theory

M.K. Pandit [a], B.N. Singh [b,*], A.H. Sheikh [c]

[a] Department of Ocean Engineering and Naval Architecture, Indian Institute of Technology, Kharagpur 721302, India
[b] Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur 721302, India
[c] School of Civil, Environmental and Mining Engineering, The University of Adelaide, SA 5005, Australia

ARTICLE INFO

Article history:
Received 25 May 2007
Accepted 11 March 2008
Available online 5 May 2008

Keywords:
Buckling load
Plate theory
Sandwich
Finite element analysis

ABSTRACT

An improved higher order zigzag theory is presented and it is applied to study the buckling of laminated sandwich plates. The present theory satisfies the conditions of transverse shear stress continuity at all the layer interfaces including transverse shear stress free conditions at the top and bottom surfaces of the plate. The variation of in-plane displacements through thickness direction is assumed to be cubic for both the face sheets and the core, while transverse displacement is assumed to vary quadratically within the core but it remains constant over the face sheets. The core is modeled as a three-dimensional elastic continuum. An efficient C^0 finite element is proposed for the implementation of the improved plate theory. The accuracy and range of applicability of the present formulation are established by comparing the present results with 3D elasticity solutions and other results available in literature.

1. Introduction

Sandwich panels possess high structural efficiency because of their excellent properties such as high strength-to-weight ratio, good energy and sound absorption capability, and often low production cost. They are mainly used in aerospace, marine and aircraft industry as thin-walled structures but at present the application of these structures has been extended to automobile, petrochemical and other industries. Structural sandwich is basically a layered composite formed by bonding two thin stiff face sheets to a thick low strength core layer. The face sheets resist nearly all of the applied in-plane and bending loads while the core carries transverse shear loads and stabilizes the outer sheets against buckling under edgewise compression, torsion or bending.

The most important feature of laminated composites and sandwich construction is that these materials are relatively weak in shear due to their low shear modulus compared to that of extensional rigidity. So, the effect of shear deformation is quite significant and it must be suitably taken care of in the analysis of such composite structures to realistically predict their behavior. In this context, the first-order shear deformation theory [1] may be considered as the starting step where uniform transverse shear strain is taken over the entire laminate thickness. As the actual variation over the laminate thickness is not uniform, it requires an arbitrary shear correction factor. In order to overcome this limitation, higher order shear deformation theories are proposed by Lo et al. [2], Reddy [3] and few others where the warping of the plate section in the transverse direction is represented by simply expanding the orders of the polynomials used for the in-plane displacements. These plate theories [1–3] are defined as single layer plate theory where the transverse shear strain is continuous across the entire thickness, which leads to discontinuity in the variation of the transverse shear stresses at the layer interfaces. But the actual situation is entirely opposite and the degree of discontinuity in the transverse shear strain is much more severe in a sandwich plate at the core face sheet interfaces due to a wide variation of their material properties.

In order to overcome the above problem, the layer-wise plate theories [4–7] have bee proposed where the unknowns are taken at each layer interface. These plate theories are sufficiently accurate as the variation of displacement components along the plate thickness can be represented appropriately. The major disadvantage of these theories is their large computational efforts since the number of unknowns increases with the increase of number of layers. In this context, the development of the refined plate theories [8–10] may be considered as a major step where the unknowns for the in-plane displacements at different layer interfaces are expressed in terms of those at the reference plane taking constant transverse displacement across the plate thickness. This is accomplished by satisfying the transverse shear stress continuity conditions at the layer interfaces. In these refined theories, the in-plane displacements have a piecewise linear

* Corresponding author. Tel.: +91 3222 283026; fax: +91 3222 255303.
E-mail address: bnsingh@aero.iitkgp.ernet.in (B.N. Singh).

0263-8231/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.tws.2008.03.002

variation across the plate thickness, which gives a zigzag pattern. A further improvement in this direction is due to Di Sciuva [11], Bhaskar and Varadan [12], Cho and Parmerter [13] and some other investigators who considered the variation of in-plane displacements to be superposition of linearly varying piecewise field on an overall higher order variation. In this context, the contribution of Cho and Parmerter [14] may be stated who have combined the concept of the Reddy's simple higher order plate theory [3] with the layer-wise zigzag plate theory [8] to provide an efficient higher order refined plate theory. The refined plate theories perform well for the analysis of sandwich plates with transversely incompressible core.

For a sandwich plate with soft core, the transverse flexibility of the compressible core may develop a face wrinkling mode in addition to the overall buckling under in-plane compressive load where the face sheets may buckle into the core region while the entire structure remains stable. Also there might be other modes of buckling depending upon the material properties of the face sheets and the core. Thus, in order to have a more accurate prediction of the buckling behavior of these structures, the effect of core transverse normal strain should be considered. Unfortunately, the variation of transverse displacement across the plate thickness can't be simply taken like that of in-plane displacements in the refined plate theories mentioned above. Though there are some attempts for the inclusion of this effect, but the performance is not found to be satisfactory with a computationally efficient model like refined plate theory having restricted number of unknowns.

So it should be noted that the conventional modeling techniques [15–17] adopted by different researcher are not adequate for the analysis of sandwich plates having a soft core. Incidentally, there are a number of studies [18–27] on sandwich plates neglecting core transverse deformability. A simple approach is adopted by Yuan and Dawe [28] to consider the effect of transverse normal deformation of the core. The laminated face sheets are simply modeled as either classical plate theory or first-order shear deformation theory while the displacements of the core are linearly interpolated from the displacements of the face sheets. A quadratic variation for the in-plane displacements over the core thickness is taken with the help of additional unknowns at the core mid-plane. Kant and Swaminathan [29] considered the effect of transverse normal strain in the buckling analysis of simply supported composite and sandwich plate but the higher order shear deformation theory adopted by them is based on single layer theory. It cannot capture the transverse shear strain discontinuity at the layer interfaces as mentioned earlier. A higher order layer-wise model has been proposed by Dafedar et al. [30] who have taken three displacements and three transverse stresses at the interfaces, which are used to have a cubic through thickness variation for the displacement components in any layer. As it involves a large number of unknowns, they have proposed a simplified model where the technique is applied to the entire plate treating it as a single layer. The simplified model is found to have inferior performance in predicting the buckling loads of sandwich plates as expected.

A much more specific study in this direction is due to Frostig [31] who proposed an efficient analytical model based on higher order sandwich panel theory to study the buckling of sandwich panels with flexible core. In this context, the three-dimensional elasticity solutions of Noor et al. [32] for buckling of simply supported sandwich panels with composite face sheets and similar studies due to Noor [33] and Srinivas and Rao [34] for stability of laminated plates are simply the bench marking studies. However, these are applicable to a specific class of problem due to analytical nature of these approaches.

Considering all these aspects in view, an attempt has been made in this investigation to develop an improved plate model to study the buckling of laminated sandwich plate with transversely flexible core. The in-plane displacement fields are assumed as a combination of a linear zigzag model with different slopes in each layer and a cubically varying function over the entire thickness. The out-of-plane displacement is assumed to be quadratic within the core and constant throughout the faces. The plate model is implemented with a computationally efficient C^0 finite element developed for this purpose and applied to solve a number of sandwich plate problems.

2. Mathematical formulation

The in-plane displacement field (Fig. 1) is taken in a similar manner as those of Cho and Parmarter [14] as follows:

$$U = u + z\theta_x + \sum_{i=1}^{n_u-1}(z - z_i^u)H(z - z_i^u)\alpha_{xu}^i + \sum_{j=1}^{n_l-1}(z - z_j^l)H(-z + z_j^l)\alpha_{xl}^j + \beta_x z^2 + \eta_x z^3 \quad (1)$$

$$V = v + z\theta_y + \sum_{i=1}^{n_u-1}(z - z_i^u)H(z - z_i^u)\alpha_{yu}^i + \sum_{j=1}^{n_l-1}(z - z_j^l)H(-z + z_j^l)\alpha_{yl}^j + \beta_y z^2 + \eta_y z^3 \quad (2)$$

where u, v denote the in-plane displacement of any point in the reference plane (plate mid-plane), θ_x and θ_y are the rotations of the normal to the mid-plane about y-axis and x-axis, respectively, n_u and n_l are number of upper and lower layers, respectively, β_x, β_y, η_x, η_y are the higher order unknowns, α_{xu}^i, α_{xl}^i, α_{yu}^i and α_{yl}^i are the change of slopes at the upper/lower ith interface between ith layer and (i+1)th layer and $H(z - z_i^u)$ and $H(-z + z_j^l)$ are the unit step functions.

The transverse displacement (Fig. 2) is assumed to vary quadratically over the core thickness and constant over the upper and lower face sheets and it may be expressed as

$$W = \begin{cases} l_1 w_u + l_2 w + l_3 w_l & \text{for core,} \\ w_u & \text{for upper faces,} \\ w_l & \text{for lower faces} \end{cases} \quad (3a)$$

where w_u, w and w_l are the transverse displacement at the top, middle and bottom layers of the core, respectively, and l_1, l_2 and l_3 are the Lagrangian interpolation functions in the thickness co-ordinate which are expressed as

$$l_1 = \frac{1}{(z_1^u - z_0)(z_1^u - z_1^l)}[z^2 - (z_0 + z_1^l)z + z_0 z_1^l],$$
$$l_2 = \frac{1}{(z_0 - z_1^u)(z_0 - z_1^l)}[z^2 - (z_1^u + z_1^l)z + z_1^u z_1^l],$$
$$l_3 = \frac{1}{(z_1^l - z_1^u)(z_1^l - z_0)}[z^2 - (z_1^u + z_0)z + z_1^u z_0], \quad (3b)$$

with $z_0 = 0$ is taken as the reference plane (Fig. 1).

The stress–strain relationship of an orthotropic layer/lamina (say kth layer) having any fiber orientation with respect to