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A parametric study of the free vibration analysis of rotating laminated cylindrical shells using the method of discrete singular convolution

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Abstract

This paper deals with the free vibration analysis of rotating laminated cylindrical shells. The analysis uses discrete singular convolution (DSC) technique to determine frequencies. Regularized Shannon's delta (RSD) kernel is selected as singular convolution to illustrate the present algorithm. The formulations are based on the Love's first approximation shell theory, and include the effects of initial hoop tension and centrifugal and coriolis accelerations due to rotation. The spatial derivatives in both the governing equations and the boundary conditions are discretized by the DSC method. Frequency parameters are obtained for different types of boundary conditions, rotating velocity and geometric parameters. The effect of the circumferential node number on the vibrational behaviour of the shell is also analysed. The analysis has been verified by comparing results with those in the literature and sufficient agreement is obtained.

Keywords: Discrete singular convolution; Kernels; Free vibration; Frequencies; Rotating cylindrical shells

1. Introduction

Real physical systems or engineering problems are often described by partial differential equations, and in most cases, their closed form solutions are difficult to establish. As a result, approximate numerical methods have been widely used to solve partial differential equations that arise in almost all engineering disciplines. The discrete singular convolution (DSC) method is a numerical method that is suitable for solving differential equations. This method was developed by Wei [1,2] in the 1999s. Since its introduction, DSC method has been applied by many authors to a variety of engineering and mathematical physics problems [3–12]. Recently, free vibration analysis of plates and conical shells has been proposed by the present author [13–15]. A good comparative accuracy of DSC and differential quadrature methods for vibration analysis of rectangular plates is presented by Ng et al. [9]. It is concluded that the DSC and GDQ methods are very suitable for vibration analysis.

Rotating circular shell structures are increasingly being used in many engineering applications like aviation, rocketry, missiles, chemical, aero-space, civil and mechanical industries. Thus, frequencies and mode shapes of such structures are important in the design of systems. As a consequence, the vibration of shell structures has been extensively studied. Chen et al. [16] applied the finite element method for rotating shells. The free vibration analysis of anisotropic cylindrical shells were studied by Bert et al. [17] using the Love's first approximation shell theory. Dong [18] considered the vibration of laminated orthotropic cylindrical shells using the Donnel's shallow shell theory. Lam and Loy [19] and Hua [20] have analyzed the free vibration of rotating laminated circular shells. Generalized differential quadrature method was presented by Hua and Lam [21] to study the free vibration of thin rotating cylindrical shells. Liew et al. [22,23] applied the Ritz and harmonic reproducing kernel particle method to the solutions of the free vibration of rotating cross-ply laminated cylindrical shells. A few studies concerning free vibration analysis of conical shells have been carried out, namely by Lim and Liew [24], Lam and Loy [25] and Lim et al. [26]. Furthermore, a number of analytical and

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numerical methods have been also studied on the vibration analysis of circular cylindrical shells [27–31].

The primary objective of this study is to give a numerical solution of vibration analysis of rotating laminated cylindrical shells via the DSC approach. In this paper, paremetric vibration analysis of rotating laminated cylindrical shell is examined. This is the first instance in which the DSC method has been adopted for free vibration analysis of rotating laminated cylindrical shells.

2. Fundamental equations

Consider a cylindrical shell rotating about its symmetrical and horizontal axis at an angular velocity ω as shown in Fig. 1. The thickness of the shell, and cone length are denoted by h and L, respectively. The cylindrical shell is referred to a coordinate system (x, θ, z) as shown in Fig. 1. The components of the deformation of the cylindrical shell with references to this coordinate system are denoted by u, v, w in the x, θ and z directions, respectively. The equilibrium equation of motion in terms of the force and moment resultants can be written by [19]

$$L_x(u, v, w) - \rho_t \frac{\partial^2 u}{\partial t^2} = 0, \tag{1a}$$

$$L_\theta(u, v, w) - \rho_t \frac{\partial^2 v}{\partial t^2} = 0, \tag{1b}$$

$$L_z(u, v, w) - \rho_t \frac{\partial^2 w}{\partial t^2} = 0, \tag{1c}$$

where

$$L_x = \frac{\partial N_x}{\partial x} + \frac{1}{R(x)}\frac{\partial N_{x\theta}}{\partial \theta} + \rho_t h \omega^2 \left[\frac{\partial^2 u}{\partial \theta^2} - R(x)\frac{\partial w}{\partial x}\right], \tag{2}$$

$$L_\theta = \frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_\theta}{\partial \theta} + \frac{1}{R}\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R^2}\frac{\partial M_\theta}{\partial \theta} + \rho_t h\omega^2 R \frac{\partial^2 u}{\partial x \partial \theta} - \rho_t h\left[\frac{\partial^2 v}{\partial t^2} + 2\omega\frac{\partial w}{\partial t} - \omega^2 v\right], \tag{3}$$

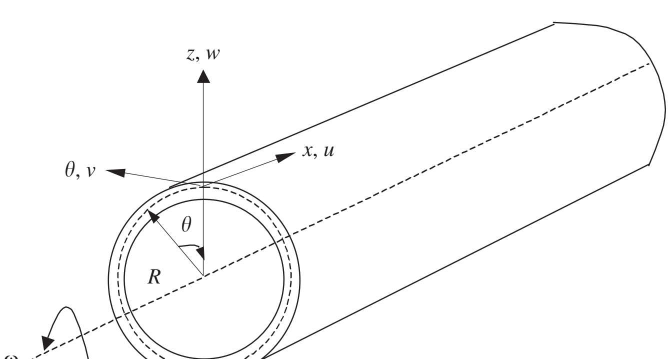

Fig. 1. Geometry of a thin rotating cylindrical shell.

$$L_z = \frac{\partial^2 M_x}{\partial x^2} + \frac{2}{R}\frac{\partial^2 M_{x\theta}}{\partial x \partial \theta} + \frac{1}{R^2}\frac{\partial^2 M_\theta}{\partial \theta^2} - \frac{N_\theta}{R} + \rho_t h\omega^2\left[\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right] - \rho_t h\left[\frac{\partial^2 w}{\partial t^2} - 2\omega\frac{\partial v}{\partial t} - \omega^2 w\right], \tag{4}$$

where

$$\rho_t(x, \theta) = \frac{1}{h}\int_{-h/2}^{h/2} \rho(x, \theta, z)\,\mathrm{d}z, \tag{5}$$

where ρ and ρ_t are, respectively, the density and density per unit area. Moment resultants and in-surface force can be obtained by [19]

$$N = (N_x, N_\theta, N_{x\theta})^{\mathrm{T}} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta})^{\mathrm{T}}\,\mathrm{d}z, \tag{6a}$$

$$M = (M_x, M_\theta, M_{x\theta})^{\mathrm{T}} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta})^{\mathrm{T}} z\,\mathrm{d}z, \tag{6b}$$

where the stress vector field $(\sigma)^{\mathrm{T}} = \{\sigma_x, \sigma_\theta, \sigma_{x\theta}\}$. Based on Love's first approximation theory [25], the strain components of this vector are defined as linear functions of the normal (thickness) coordinate z, namely

$$\varepsilon_x = \varepsilon_1 + z\kappa_1, \quad \varepsilon_\theta = \varepsilon_2 + z\kappa_2, \quad \varepsilon_{x\theta} = \gamma + 2z\tau, \tag{7}$$

where $\{\varepsilon\}^{\mathrm{T}} = \{\varepsilon_1, \varepsilon_2, \gamma\}$ and $\{\kappa\}^{\mathrm{T}} = \{\kappa_1, \kappa_2, 2\tau\}$ are, respectively, the strain and curvature vectors of the reference surface. They are defined by [26]

$$\varepsilon_1 = \frac{\partial u}{\partial x}, \tag{8}$$

$$\varepsilon_2 = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right), \tag{9}$$

$$\gamma = \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}, \tag{10}$$

$$\kappa_1 = -\frac{\partial^2 w}{\partial x^2}, \tag{11}$$

$$\kappa_2 = -\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \theta}, \tag{12}$$

$$\tau = -\frac{2}{R}\frac{\partial^2 w}{\partial x \partial \theta} + \frac{\partial v}{\partial x}. \tag{13}$$

In Eq. (9), N and M are the force and moment resultants and can be obtained by

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} e \\ \kappa \end{Bmatrix}, \tag{14}$$

where A_{ij}, B_{ij} and D_{ij} are the extensional, coupling and bending stiffnesses and calculated from the following equations:

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}^*(1, z, z^2)\,\mathrm{d}z, \quad i = 1,2 \quad j = 3 + i. \tag{15a}$$