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Sensitivity analysis of the elastic buckling of cracked plate elements under axial compression

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ABSTRACT

Cracks of any size may occur at various locations and orientations throughout thin-walled structures. The presence of cracks in such structures can considerably affect their load bearing behaviour. This paper addresses a finite element study on the buckling strength of a cracked plate with simple supports subjected to an axial compressive edge load. The effects of crack location, crack orientation, crack length and plate aspect ratio are analysed. The size of the crack as well as its location and orientation are shown to have significant effects on the buckling behaviour of the plate under compressive loading. Some new results are also discussed in detail.

1. Introduction

Plates are the most common structural elements used in the majority of thin-walled structures, dealing with the fields of naval architecture, civil, mechanical, and aerospace engineering. Ships and offshore structures are some examples of intricate thin-walled structures that consist of plate elements, which are subjected to a variety of load combinations. An ageing thin-walled structure is vulnerable to various types of defects and damages induced by different phenomena such as corrosion and fatigue cracking. It is of crucial importance, from many design and safety aspects, to study and understand both behaviour and strength of such plate elements in intact, defected and damaged conditions. The importance is well understood on the failure events that have led to the loss of life.

The strength characteristics and behaviour of plates with a crack have received some attention by numerous investigators in recent years. Some of the researchers have studied the problem under tensile loads [1–6]. Brighenti [5,6] in his latest works has calculated the critical load multiplier for a cracked plate in tension. In his studies, some parameters were changed and finally an approximate theoretical model to explain and predict the buckling phenomena in cracked plates under tension was proposed. Crack influences on the vibration and parametric instability of plates were also studied by some of the researchers [7–11]. Paik et al. in his extensive work [12] made a numerical/experimental study on the collapse behaviour of plates with crack under both tensile and compressive loads. They finally derived the ultimate strength formulations for such cases. The influence of central cracks on buckling and post-buckling behaviour of shear panels was studied by Alinia et al. [13]. They have reached the conclusion that the mesh density at crack tips exceptionally plays a dominant role in the numerical accuracy, but the vicinity of crack sides may have the mesh refinement similar to uncracked panels.

Besides, the problem of cracked plate elements under compression was the subject of few research studies [1,5,6,10,11,14]. Most of these studies were performed on the plates with either central or perpendicular-to-one-edge crack of varying crack length. A crack in a plate element may be generally observed at any location and orientation. The initiation of such a crack may be due to fatigue, impact, imperfections and so on. No extensive studies have been published yet, according to the authors' knowledge and literature survey, on the buckling analysis of cracked plate elements under compression, including all affecting parameters such as crack location, orientation and size. In this regard, the objectives of the present study are: (i) to investigate the buckling strength characteristics of a cracked plate under monotonically applied axial compression and (ii) to study the sensitivity of the buckling strength of cracked plate elements under axial compression to the crack size, location and orientation.

To do so, a series of eigenvalue buckling analysis on cracked plate elements under axial compression is carried out using an

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Nomenclature

a plate length
b plate width
c crack length
$D = Et^3/(12(1-\nu^2))$ plate flexural rigidity
E Young's modulus
J determinant of *Jacobian* matrix
k, $k^{Uncracked}$, $k^{Cracked}$ Buckling coefficient in general and in case of uncracked and cracked plates, respectively
N_x membrane forces per unit length of the plate along x-axis
N_y membrane forces per unit length of the plate along y-axis
N_{xy} shearing forces per unit length of the plate along the edges of the plate
N_i shape functions ($i = 1$–8)
P_{CR} critical buckling load
$P_{cr}(i)$ critical buckling load of the ith mode
$\dot{P}_{cr}(1)$ critical buckling load of the first mode of the uncracked plate
t plate thickness
u movement of the node along x-axis
U internal strain energy
u_x, u_u, u_z displacements along x-axis, y-axis, z-axis, respectively
v movement of the node along y-axis
W external work
w or $w(x,y)$ plate elastic surface (transverse displacements) or out-of-plane displacements
X_c, Y_c the x-coordinate and y-coordinate of the mid point of the crack, respectively
$\beta = (b/t)\sqrt{(\sigma_Y/E)}$ plate slenderness parameter
$\gamma = a/b$ plate aspect ratio
ν Poisson's ratio
θ orientation angle
λ load scalar multiplier
λ_{cr} smallest level of external load for which there is a bifurcation
σ_a applied in-plane stresses along x-direction or edge with length of a
σ_{CR}, $(\sigma_a)_{CR}$ critical buckling strength
${\sigma_{CR}}^{Uncracked}$, ${\sigma_{CR}}^{Cracked}$ critical buckling strength respectively for the uncracked and cracked plate, respectively
σ_Y material yield stress
$[B]$ strain-nodal deflection relationship matrix
$\{d\}$ vector of nodal deflections
$\{\delta D\}$ buckling displacements or eigenvector
$\{D\}_{ref}$ displacements of the reference configuration
$[E]$ stress–strain relationship matrix
$[J]$ *Jacobian* matrix
$\{R\}_{ref}$ reference load vector
$\{R\}$ load vector
$[K]$ conventional stiffness matrix
$[K_\sigma]_{ref}$ stress stiffness matrix corresponding to the reference load vector $\{R\}_{ref}$
$[K_\sigma]$ stress stiffness matrix corresponding to load vector $\{R\}$
$[K]_{net}$ total or net stiffness
$f_{,x} = \partial f/\partial x$derivative of function f (in general) with respect to x

in-house finite element (FE)-based program. Due attention has been focused on FE modelling of the problem and the significance of various parameters such as the crack size, location and orientation. The research findings by Javidruzi and his collaborators in their earlier works [10,11], are incorporated into the FE modelling of the problem. The present study does not aim to address crack propagation scenarios and fracture related to the critical crack length considerations.

2. Formulation

Based on the von Kármán theory of plates [15], the fourth-order partial differential equation which describes the plate's deflection w when considering the second-order geometrical effects (or membrane loadings), is

$$\nabla^4 w = -\frac{1}{D}\left(N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y}\right) \tag{1}$$

Fig. 1 shows a simply supported plate under axial in-plane compression σ_a along the x-axis. In such cases, $N_x = -\sigma_a t$ and $N_y = N_{xy} = 0$. Therefore, the general governing equation of plates, Eq. (1), reduces to the following form:

$$\nabla^4 w = -\frac{\sigma_a t}{D}\frac{\partial^2 w}{\partial x^2} \tag{2}$$

Since the edges are simply supported, the deflected shape of the plate can be conveniently expressed by a sinusoidal form

$$w = \sum_i\sum_j C_{ij}\sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right) \tag{3}$$

Eq. (3) satisfies both the boundary conditions and the general bi-harmonic equation, where Eq. (2) is a special case. Assuming a free-to-move-inward condition at the edge of the plate under in-plane compression, there is no longitudinal strain produced in the mid-plane of the plate and thus, the internal strain energy, due to bending only, can be expressed in the following form:

$$U = \frac{D}{2}\int_0^a\int_0^b\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\}\mathrm{d}x\,\mathrm{d}y \tag{4}$$

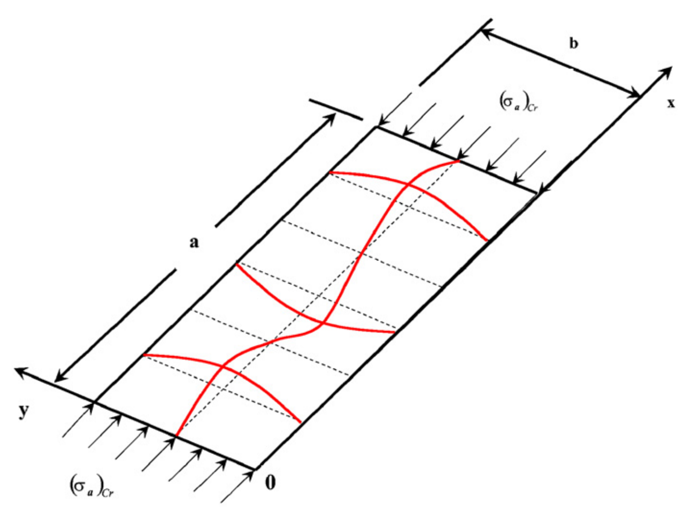

Fig. 1. Buckled shape of a long uncracked simply supported plate.