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Thermoelastic solution of a functionally graded variable thickness rotating disk with bending based on the first-order shear deformation theory

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ABSTRACT

A theoretical solution for thermoelastic analysis of functionally graded (FG) rotating disk with variable thickness based on first-order shear deformation theory (FSDT) is presented. Material properties and disk thickness profile are assumed to be represented by power law distributions. A semi analytical solution for displacement field is given under two types of boundary conditions applied for solid and annular disks. The effects of the material grading index and the geometry of the disk on the stress and displacement fields are investigated. Mechanical responses homogeneous disks versus FG disks are compared and verified with the known results in the literature. It is seen that the transverse displacements in FG solid disks with roller support condition at the outer surface remain between the minimum displacement value for the full-ceramic disk and the maximum displacement value for the full-metal disk. It is found that the transverse displacements in FG mounted disks with free condition at outer surface may not lie in between the displacement values for full-metal and full-ceramic disks. It is observed that the absolute moment resultant for FG mounted disk with concave profile is lowest compared to the FG mounted disk with linear or convex profile. It can be concluded that the gradation of the metal–ceramic components and the geometry of the disk are significant parameters in the thermomechanical responses of FG disks.

1. Introduction

Functionally graded materials (FGMs) are microscopically inhomogeneous composite materials, in which the volume fraction of the two or more materials is varied smoothly and continuously as a function of position along certain dimension(s) of the structure from one point to the other [1,2]. These materials were first introduced in 1984 in Japan as ultra light temperature-resistant materials for space vehicles [3]. They are usually made of a mixture of ceramic and metals. The ceramic constituent of the material provides the temperature resistance due to its low thermal conductivity, the metal constituent, on the other hand, helps prevent fracture caused by thermal stress [4]. In the following, we mention some FGM related literature.

Noda and Tsuji [5,6] discussed steady-state thermal stresses in a plate made of FGMs with temperature-related material properties. They determined an optimal FGM to minimize steady-state thermal stresses. Tanigawa [7] solved a one-dimensional transient heat conduction problem and obtained the associated thermal stress in a nonhomogeneous plate. He further formulated the optimum material composition that minimized the thermal stress. Tanigawa et al. [8] examined the transient thermal stress distribution of FGM plates induced by unsteady heat conduction with temperature-dependent material properties. Zimmerman and Lutz [9] derived an exact solution for the problem of uniformly heating FG cylinder whose elastic modulus and thermal expansion coefficient varied linearly with radius. Jabbari et al. [10] gave a general thermoelastic analysis of one-dimensional steady-state hollow thick cylinder made of FGM. The temperature distribution was assumed to be a function of radius. Eslami et al. [11] presented a general solution for one-dimensional steady-state thermal and mechanical stresses in a hollow thick sphere made of FGM. The temperature distribution was assumed to be a function of radius. Shao and Ma [12] presented thermo-mechanical analysis of FG hollow circular cylinders subjected to mechanical loads and linearly increasing boundary temperature. Apart from studies on thermoelastic analysis as mentioned above, many authors added bending effects to thermoelastic problems. Some of the important references are mentioned below.

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The linear thermal bending of FGM plate in steady state and stress analysis in transient heat conduction with temperature dependent material properties was studied by Tanigawa et al. [8]. Reddy and Chin [13] solved the heat conduction and the thermoelastic equations for a FG axisymmetric cylinder subjected to thermal loading. They also formulated a thermoelastic boundary value problem for a FG plate using the first-order shear deformation theory (FSDT) that accounted for the transverse shear strains and the rotations coupled with a three-dimensional heat conduction equation. Reddy et al. [4] studied the axisymmetric bending and stretching of FG solid and annular circular plates using the FSDT and found the solutions for deflections, force and moment resultants in terms of the corresponding quantities of isotropic plates based on the classical Kirchhoff plate theory. Cheng and Batra [14] related the deflections of a simply supported FG polygonal plate given by the FSDT and a third-order shear deformation theory (TSDT) to an equivalent homogeneous Kirchhoff plate. Cheng [15] presented a new set of nonlinear equations for a rectangular inhomogeneous plate in the sense of the von Karman large deflection using the displacement and stress potential functions based on the FSDT. They solved these equations associated with boundary conditions by using mixed Fourier series technique. Lanhe [16] derived equilibrium and stability equations for a moderately thick rectangular plate made of FGMs under thermal loads based on the FSDT and solved them analytically assuming material properties given by a power law of thickness. Ganapathi and Prakash [17] investigated the thermal buckling of a simply supported FG skew plate using FSDT in conjunction with the finite element approach and assuming linear and nonlinear forms of temperature-rise across the thickness. Arciniega and Reddy [18] presented a nonlinear analysis of FG shells geometrically. The FG shell consisting of two constituents, i.e., ceramic and metal was assumed to be graded through the thickness from one surface of the shell to the other and a tensor-based finite element formulation with curvilinear coordinates and FSDT was used to develop the finite element of the FG shell. Bayat et al. [19] used FSDT and von Karman theories and developed a new set of equilibrium equations with small and large deflections for a FG rotating disk with axisymmetric bending and steady-state thermal loading assuming material properties of the disk varying in the thickness direction.

Many studies conducted on FGMs have been related to the analysis of thermal stress and deformation (see e.g. [20–24] and the references therein). Ruhi et al. [24] presented a semi-analytical thermoelastic solution for finitely long thick-walled cylinder made of FGMs. Durodola and Attia [25,26] presented a finite element analysis for FG rotating disks using commercial software package. The disks were modeled as non-homogeneous orthotropic materials such as those obtained through non-uniform reinforcement of metal matrix by long fibers. Three types of gradation distributions for the Young's modulus E were considered in the hoop direction relative to matrix material modulus. Kordkheili and Naghdabadi [27] presented semi-analytical thermoelastic solution for hollow and solid rotating axisymmetric disks made of FGMs under plane stress condition. They compared their results with those of Durodola and Attia [25,26] under the centrifugal loading.

Many earlier studies on rotating disks (see e.g., Tutuncu [28] and the references there in) have considered disks with uniform thickness. However, in later works several authors have emphasized the importance of variable thickness in the rotating disks [29–32]. Recently, Eraslan and Orcan [33] and Orcan and Eraslan [34] found that the stresses in rotating annular or solid disks with variable thickness were much lower than those with uniform thickness at the same angular velocity.

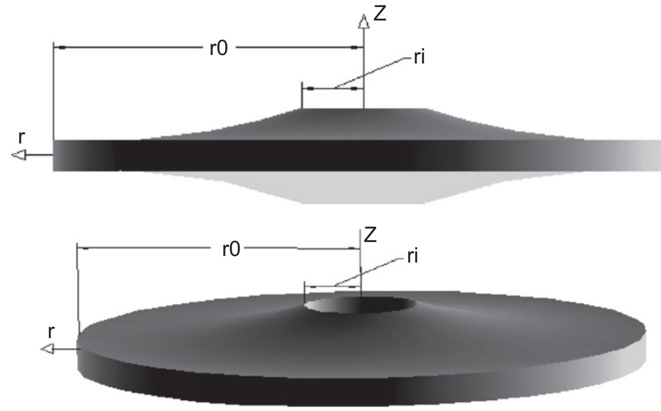

Fig. 1. Configuration of a thin disk with variable thickness.

To the best of the authors' knowledge, no work has been reported till date that analyzed FG disks with variable thickness under thermal, body and bending loads. This very fact motivates the investigation of the present study. To be more specific, in this paper a thin FG disk of variable thickness (Fig. 1) subjected to thermal, body and bending loads is considered. The mechanical loading is obtained by having bending load in the z direction, thermal and centrifugal loads in the radial direction. The material properties of the constituent components of the disk are assumed to be represented by a power law distribution along the radial direction of the disk. The FSDT is used. This work aims to investigate the effects of some basic factors such as material property gradation, the thickness profile of the disk and boundary conditions on the displacement and stress fields. Solid disk with roller-support at the outer edge $r = r_o$ and the hollow disk with clamped support at the inner edge $r = r_i$ and free at outer edge $r = r_o$ are considered. In both cases, semi-analytical solutions for the normalized displacement and force resultant components are obtained.

In the semi-analytical method, the radial domain is divided into some virtual sub-domains in which the mechanical property is assumed to be constant. This assumption yields the governing equations in each sub-domain as ordinary differential equations with constant coefficients whose general solution can be written involving certain unknowns. These unknowns can be determined as solution of a set of linear algebraic equations imposing the continuity condition of displacement and radial stress at the interface of the adjacent sub-domains together with global conditions. Increasing the number of sub-domains (divisions) in the radial direction increases the accuracy of the solution.

2. Gradation relation

In this study, the property variation $P(r)$ of the material in the FG disk along the radial direction is assumed to be of the following form [13,21]:

$$P(r) = (P_o - P_i)\left(\frac{r - r_i}{r_o - r_i}\right)^n + P_i; \quad r_i < r < r_o \tag{1a}$$

Here P_o and P_i are the corresponding properties of the outer and inner faces of the disk respectively; r_o and r_i are the outer and inner radii of the disk respectively; $n \geqslant 0$ is the volume fraction exponent (also called grading index in this paper). The power-law (1a) is widely accepted and it reflects a simple rule of mixtures in terms of the volume fraction of the materials. In this study, the Poisson's ratio ν is assumed to be constant and the elastic modulus E, thermal conductivity K, thermal coefficient of