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GBT formulation to analyse the first-order and buckling behaviour of thin-walled members with arbitrary cross-sections

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ABSTRACT

This paper presents the derivation, validation and illustration of a generalised beam theory (GBT) formulation intended to perform first-order and buckling analyses of arbitrary thin-walled members, namely members with cross-sections that combine closed cells with open branches. Following a brief overview of the so-called "conventional GBT formulation", as well as of the available extensions for different specific cross-section types, the paper addresses in detail the modifications that must be incorporated into the GBT cross-section analysis procedure to handle the simultaneous presence of closed cells and open branches. The proposed formulation is then employed to analyse the first-order and buckling behaviours of thin-walled members (mostly beams) with complex cross-sections. For validation purposes, the GBT-based numerical results are compared with values yielded by shell finite element and finite strip analyses.

1. Introduction

Generalised beam theory (GBT) was originally developed by Schardt [1–3] and may be viewed as an extension of Vlasov's classical prismatic bar theory that accounts for both cross-section out-of-plane (warping) and in-plane deformation. In GBT, the deformed configuration or buckling/vibration mode shape of a given member is expressed as a linear combination of pre-determined cross-section deformation modes with longitudinally varying amplitudes — for illustrative purposes, Fig. 1 shows the first seven deformation modes of a hat-section. This quite unique modal feature renders the application of GBT considerably more versatile and efficient than the use of "equivalent" (similarly accurate) finite strip or shell finite element models. Indeed, it has been shown that GBT constitutes a powerful, elegant and clarifying tool to solve structural problems involving prismatic thin-walled members (e.g., [4–6]).

However, the vast majority of the research work devoted to GBT was done in the context of thin-walled members with either open unbranched or closed single-cell cross-sections (see Fig. 2(a) and (c_1)). For open branched cross-sections, like the ones shown in Fig. 2(b), although specific GBT applications had been previously addressed by Möller [7], Mörschardt [8] and Dégée & Boissonnade [9], it was only quite recently that a more general treatment of this type of cross-section was developed and validated by Dinis et al. [10]. For multi-cell cross-sections, it is possible to employ the shear deformable formulation developed by Möller [7], which bears considerable resemblance with Vlasov's General Variational Method[1] [11]. Although Möller's formulation can also be applied to *arbitrary* cross-section shapes (cross-sections combining closed cells with open branches—see Fig. 2(d_1)–(d_2)), such application is by no means straightforward (details are provided in the next sections) and often involves an unnecessarily large number of deformation modes, which makes it computationally less efficient—moreover, it may also be argued that it somewhat deviates from the "spirit" of Schardt's work.

The aim of this paper is to derive, validate and illustrate the application of a GBT formulation to perform linear (first-order) and buckling analyses of folded-plate thin-walled members with fully arbitrary cross-section shapes, namely those combining any number of closed cells (with or without common walls) with an arbitrary amount of "open branches" (i.e., walls not belonging to any closed sub-section)—such branches may either exhibit a free end or connect adjacent closed cells (see Fig. 2(d_2)).

Initially, one presents a brief overview of the available GBT formulations, which make it possible to analyse (i) open unbranched members [2,3],[2] (ii) open branched members [10], (iii) closed

[1] This method is also known as the "Generalised Coordinate Method" (see, for instance, [12]).
[2] Henceforth designated as "conventional GBT".

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Nomenclature

B, **C**, **D**, **X**	GBT cross-section modal matrices
x, y, z	wall mid-surface local axes
S	cross-section mid-line
U_x, U_y, U_z	wall displacement components along the local axes
u, v, w	wall mid-surface displacement components along the local axes
$\bar{u}_k$, $\bar{v}_k$, $\bar{w}_k$	deformation mode shape functions
ϕ_k	displacement mode amplitude functions
n	number of deformation modes
k, i	indices varying between 1 and n
E, G, ν	material properties (Young's and shear moduli, Poisson's ratio)
t	wall thickness
$\boldsymbol{\sigma}$	stress vector
$\boldsymbol{\varepsilon}$	strain vector
$(\cdot)^M$	membrane term
$(\cdot)^B$	bending term
E	elastic constitutive matrix

single-cell members [2,13] and (iv) closed multi-cell members [7]. Then, one addresses the modifications that must be incorporated into the GBT formulation in order to (i) retain the "spirit" of Schardt's original work and (ii) to be able to handle members exhibiting simultaneously closed cells and open branches. These modifications concern mainly issues related to the performance of the so-called "GBT cross-section analysis", namely the choice of the most appropriate warping and shear "elementary functions". Finally, the efficiency and versatility of the proposed GBT formulation are illustrated through its application to determine (i) the first-order behaviour of two steel cantilevers with a "bridge deck"-type cross-section and acted by an eccentric vertical load and (ii) the buckling behaviour of simply supported channel and I-section steel girders exhibiting closed cells and subjected to uniform compression and/or bending. For validation purposes, the GBT-based results are compared with values yielded by shell finite element and semi-analytical finite strip analyses performed in the codes ADINA [14] and CUFSM$_{2.6}$ [15].

2. An overview of the conventional GBT analysis

The so-called "conventional GBT" is intended to analyse the behaviour of elastic isotropic prismatic thin-walled members with *open unbranched* cross-sections (see Fig. 2(a)). Its application involves the performance of two main tasks, namely (i) a "cross-section analysis", which concerns the identification of the deformation modes and the evaluation of the associated modal mechanical properties, and (ii) a member (first-order, buckling, vibration, etc.) analysis, in which the appropriate differential equilibrium equations must be solved (e.g., [2,4–6]). In the next paragraphs, the main concepts and procedures related to performing GBT analyses are briefly reviewed.

Consider the arbitrary member depicted in Fig. 3, where x, y, z are local coordinates corresponding to each wall length, cross-section mid-line and thickness, respectively. Following the traditional GBT kinematic description [2], the wall displacement vector, expressed in the local coordinates, is given by

$$\mathbf{U} = \begin{Bmatrix} U_x \\ U_y \\ U_z \end{Bmatrix} = \begin{Bmatrix} u - zw_{,x} \\ v - zw_{,y} \\ w \end{Bmatrix}, \tag{1}$$

where Kirchhoff's plate assumption is adopted, the commas indicate differentiation and u, v, w are the wall mid-surface displacement components measured along the local axes x, y, z,

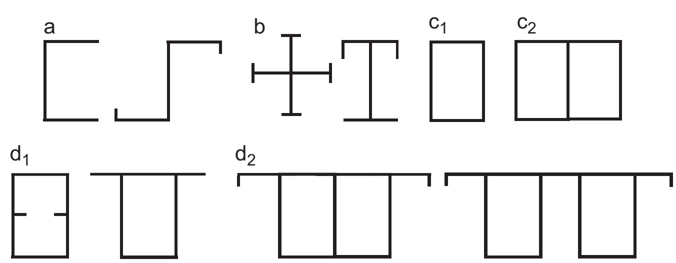

Fig. 2. Cross-section types: (a) open unbranched, (b) open branched, (c) closed single and multi-cell and (d) closed cells with open branches.

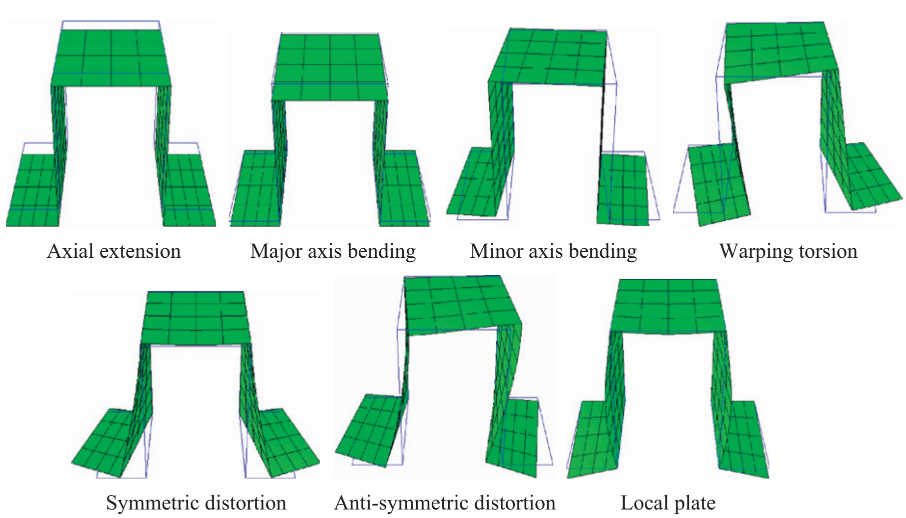

Fig. 1. First seven GBT deformation modes of a hat-section.