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Stability analysis of composite plates under localized in-plane load

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ABSTRACT

Stability characteristics of isotropic and composite plates subjected to partial or concentrated compressive edge loads are investigated here using a four-noded shear-flexible quadrilateral high-precision plate-bending element. A complete cubic polynomial shape function is used to interpolate the in-plane displacements for better accuracy in capturing the high-stress zone near the locally distributed edge load. The influences of location and distribution of in-plane edge compression on the buckling load of composite plates are studied. The nonlinear governing equations based on von Kármán's assumptions are solved by Newton–Raphson technique to get the hitherto unreported postbuckling equilibrium paths of partially loaded plates made of isotropic, symmetric, and unsymmetric laminates. Marguerre's shallow-shell theory is employed to study the effect of sinusoidal imperfection on the nonlinear behavior of composite plates under partial in-plane load.

1. Introduction

Composite plates and shells are increasingly used in thin-walled structural components of aircrafts, submarines, automobiles, and other high-performance application areas. These structural components are sometimes subjected to uniform or non-uniform compressive in-plane load and may become unstable. Hence, the knowledge of the stability characteristics of such structural components under uniform, partial, or localized edge load is essential for their reliable and light-weight design. It is observed from the existing literature [1–3] that the buckling and postbuckling behavior of rectangular composite plates subjected to uniform edge load have been extensively investigated, while studies on the stability characteristics of such plates under localized (partial or concentrated) edge loads are limited.

A few analytical [4–7] and numerical [8–10] investigations on the critical buckling loads of isotropic plates under non-uniform edge loads are available in the literature. Baker and Pavolic [4] employed Ritz method and Brown [5] applied the conjugate load displacement method to study the stability characteristics of plates under localized in-plane load, while, Leissa and Ayoub [6] used Ritz method and Kaldas and Dickinson [7] applied Raleigh–Ritz method to study the vibration frequencies and buckling loads of isotropic plates subjected to partial edge load. Calculation of buckling load of rectangular plates under localized in-plane load with or without localized in-plane support requires accurate estimation of non-uniform pre-buckling stresses from a two-dimensional plane-stress analysis. Hence, numerical methods, such as finite element procedure, may be more proficient over the analytical methods while studying such problems. Recently, Liew and Chen [8] used radial point interpolation method to study the buckling behavior, Srivastava et al. [9] employed nine-noded isoparametric element to investigate the buckling and vibration characteristics, whereas, Deolasi and Datta [10] applied eight-noded quadratic element to examine the dynamic stability characteristics of shear-deformable isotropic plates under partial edge compression. However, studies on the stability characteristics of composite plates under partial edge load are scarce in the literature.

Most recently, Sundaresan et al. [11] studied the critical buckling loads of composite plates under partial in-plane load using an eight-noded quadratic isoparametric element, while, Chakrabarti and Sheikh [12,13] developed a six-noded triangular element to investigate the linear buckling loads of laminated composite and laminated sandwich plates under partial edge compression. However, to the best of the authors' knowledge, postbuckling analysis of isotropic or composite plates subjected to partial or concentrated in-plane edge load with partial in-plane support is not yet commonly available in the literature.

In the present work, a four-noded shear-flexible quadrilateral high-precision shallow-shell element is employed to study the buckling and postbuckling characteristics of composite plates under concentrated or partial in-plane load. A complete cubic polynomial shape function is used to interpolate in-plane displacements for better accuracy in capturing the non-uniform pre-buckling stresses. The nonlinear governing equation is solved by Newton–Raphson technique to determine (1) pre-buckling deformation and stresses, (2) buckling load, and (3) postbuckling

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equilibrium paths. The influence of localized edge compression on the critical buckling loads of simply supported thin and thick rectangular composite plates is examined. Thereafter, a limited parametric study is carried out to investigate the postbuckling paths of isotropic and composite perfect/imperfect square plates under partial edge loads.

2. Finite element formulations

The displacement components at a generic point (x, y, z) of a shear-deformable quadrilateral plate can be expressed as [14,15]

$$u(x,y,z) = u_0(x,y) + z\{-w_{,x} + \gamma_x(x,y)\},$$
$$v(x,y,z) = v_0(x,y) + z\{-w_{,y} + \gamma_y(x,y)\},$$
$$w(x,y,z) = w_0(x,y) \tag{1}$$

Here, u_0, v_0, w are the mid-surface displacements; γ_x and γ_y are the rotations due to shear; $()_{,x}$ and $()_{,y}$ represent the partial differentiation with respect to x and y, respectively; $\phi_x = -w_{,x}+\gamma_x(x, y)$ and $\phi_y = -w_{,y}+\gamma_y(x,y)$ are the nodal rotations.

Following Marguerre von Kármán strain–displacement relation [16], the in-plane and shear strains of a shallow shell can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ v_{0,x} + u_{0,y} \end{Bmatrix} + \begin{Bmatrix} \bar{w}_{,x} w_{,x} \\ \bar{w}_{,y} w_{,y} \\ \bar{w}_{,x} w_{,y} + \bar{w}_{,y} w_{,x} \end{Bmatrix} + \begin{Bmatrix} w_{,x}^2/2 \\ w_{,y}^2/2 \\ w_{,x} w_{,y} \end{Bmatrix} + z \begin{Bmatrix} -w_{,xx} + \gamma_{x,x} \\ -w_{,yy} + \gamma_{y,y} \\ -2.w_{,xy} + \gamma_{y,x} + \gamma_{x,y} \end{Bmatrix} = \{\varepsilon_{\mathrm{m}}\} + z\{\kappa\} \tag{2a}$$

and

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \gamma_x \\ \gamma_y \end{Bmatrix} \tag{2b}$$

Here, $\bar{w}$ represents initial (strain free) deformation of the mid-surface. It may be noted here that the second term of Eq. (2a) is added to the strain–displacement relationship of Ref. [15] to include the shallow-shell effect (imperfection).

The membrane stress resultants $\{N\}$, bending stress resultants $\{M\}$, and shear stress resultants $\{Q\}$ are expressed as

$$\{N\} = \{N_{xx}, N_{yy}, N_{xy}\}^{\mathrm{T}} = [A_{ij}]\{\varepsilon_m\} + [B_{ij}]\{\varepsilon_{\mathrm{b}}\},$$
$$\{M\} = \{M_{xx}, M_{yy}, M_{xy}\}^{\mathrm{T}} = [B_{ij}]\{\varepsilon_m\} + [D_{ij}]\{\varepsilon_{\mathrm{b}}\},$$
$$\{Q\} = [S]\{\gamma\} \tag{3}$$

where $[A]$, $[B]$, $[D]$, and $[S]$ are extensional, extension-bending, bending, and shear stiffness coefficients, respectively. For a composite laminate of thickness h, comprising of N layers with stacking angles θ_i ($i = 1,2,\ldots,N$) and layer thicknesses h_i ($i = 1,2,\ldots,N$), the necessary expressions to compute the stiffness coefficients are available in the literature [17].

Following standard procedure (minimization of potential energy), the nonlinear finite element equation and linear stability equation for the plate under in-plane compressive load may be written as

$$\left[\boldsymbol{K}_{\mathrm{L}} + \frac{1}{2}\boldsymbol{N}_1(\delta) + \frac{1}{3}\boldsymbol{N}_2(\delta,\delta)\right]\{\delta\} = \{F\} \tag{4}$$

$$[\boldsymbol{K}_{\mathrm{L}} + \lambda \boldsymbol{K}_{\mathrm{G}}]\{\delta\} = \{0\} \tag{5}$$

where $\boldsymbol{K}_{\mathrm{L}}$ is the linear stiffness matrix, $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$ are the nonlinear stiffness matrices, $\boldsymbol{K}_{\mathrm{G}}$ is the geometric stiffness matrix due to unit in-plane load, $\{\delta\}$ is the vector of nodal displacements, and $\{F\}$ the vector of nodal forces.

3. Solution procedure

The stability characteristics of composite plates are studied in three phases to obtain the pre-buckling deformation/stresses, buckling load, and postbuckling equilibrium path.

3.1. Step 1—pre-buckling analysis

Initial pre-buckling displacement is calculated by linear analysis $[\boldsymbol{K}_{\mathrm{L}}]\{\delta\} = \{F\}$ under unit in-plane load "P" (Figs. 1 and 2). Thereafter, the pre-buckling stresses and stress resultants (N_{xx}, N_{yy}, N_{xy}) are calculated using Eq. (3).

3.2. Step 2—buckling analysis

The geometric stiffness matrix $\boldsymbol{K}_{\mathrm{G}}$ corresponding to the stress resultants (N_{xx}, N_{yy}, N_{xy}) is calculated and the critical buckling load at which Euler type of bifurcation occurs is obtained from the eigenvalue Eq. (5).

3.3. Step 3—postbuckling analysis

After buckling, equilibrium equation (4) is solved by Newton–Raphson technique to obtain the postbuckling equilibrium paths of perfect/imperfect rectangular composite plates. In the absence of any imperfection, isotropic or symmetrically laminated rectangular composite plates exhibit bifurcation buckling and hence to initiate buckling, a load perturbation (uniformly distributed transverse load) that produces an approximate central displacement of 0.002h is applied. This perturbed nonlinear analysis ensures correct distribution of nonlinear stresses inside the plate.

A C^1-continuous four-noded quadrilateral plate-bending element with 14 degrees of freedom per node, namely u_0, $u_{0,x}$, $u_{0,y}$, $u_{0,xy}$, v_0, $v_{0,x}$, $v_{0,y}$, $v_{0,xy}$, w, $w_{,x}$, $w_{,y}$, $w_{,xy}$, γ_x, and γ_y is employed to study the stability characteristics of composite plates subjected to non-uniform in-plane stresses. A complete cubic polynomial

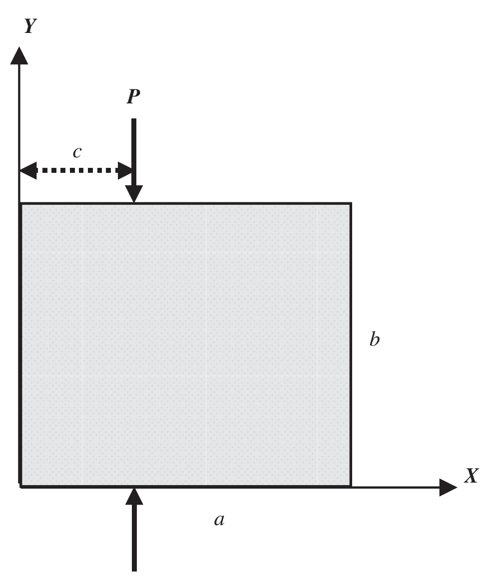

Fig. 1. A rectangular plate of size $a \times b$ is under a pair of concentrated load P acting at a distance "c" from the left edge.