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Finite element analysis of nonlinear shell structures with uncertain material property

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Abstract

The statistical dynamic responses of geometrically nonlinear shell structures with stochastic Young's modulus of elasticity are investigated in this study. In general, large deformation of the shell structures must be considered when shell structures are under excessive loading, and then the governing equations of the shell structures become nonlinear since the stiffness matrix of the system is related to the deflection. In this paper, the stochastic finite element method along with the perturbation technique is used to deal with statistical responses of shell structures with structural randomness; in particular, the Newton–Raphson iteration procedure in conjunction with Newmark scheme is adopted to solve the nonlinearity of the dynamic governing equation of shell structures. Some results obtained by the perturbation technique and those from the Monte Carlo simulation approach show a good agreement. Finally, it should be emphasized that these statistically dynamic responses are very useful for estimating the reliability of structures.

Keywords: Nonlinear shell structures; Stochastic finite element method; Perturbation technique; Newton–Raphson method; Newmark scheme; Monte Carlo simulation

1. Introduction

The geometry and material properties of structures were considered as deterministic despite the fact that they intrinsically contain uncertainty or randomness, in early studies. Recently, this discrepancy has been modified and improved by the stochastic finite element analysis, which has been studied and developed by many researchers. Among others, the following researchers have already contributed to the development of this field. Astill, et al. [1] used the Monte Carlo simulation method to study structural response variability due to random properties in structures. Hisada and Nakagiri [2–6] used the perturbation method with finite element method to investigate some structural problems. The stochastic finite element method was adopted by Baecher and Ingra [7] to predict the settlement of structure on foundation. Vanmarcke and Grigoriu [8] also used the stochastic finite element method to demonstrate the use of spatial averaging of random fields to simple beams with random rigidity. A similar approach has been used by Liu et al. [9] with applications to elasto-plastic and nonlinear dynamic problems. Shinozuka and Dasgupta [10] have adopted Neumann expansion with Monte Carlo simulation to demonstrate advantages of the formulation to dynamic problems. Also, Der Kiureghian and Ke [11] used the stochastic finite element method to study the safety and reliability of structures. Shinozuka and Co-workers [12–15] used the finite element method in conjunction with Neumann expansion and Monte Carlo simulation techniques to investigate the statistical dynamic response of structures due to random material properties or geometries in structures and studied the structural safety and reliability. In addition, Bucher and Shinozuka [16] and Kardara et al. [17] used the Green's function formulation to determine the mean square response of statically indeterminate structures. Chang and Yang [18] formulated

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a geometrically nonlinear stochastic thin-plate finite element to study the reliability of laminated plates with structural uncertainties under random in-plane loads. Chang and Chang [19] used the perturbation technique in conjunction with the finite element method to obtain statistical dynamic responses of a nonuniform beam with material randomness.

Thin shell structures are popular and widely used in many civil engineering and mechanical structures. In the design of thin shell structures, it is often necessary to consider the effect of large deflections when the structure is under excessive loading. In general, when considering the influence of large deflection of a shell structure, the relationship between strain and displacement is nonlinear; similarly, the stiffness matrix in the governing equations of the shell structure, which depends on the deflections, is nonlinear as well. These raise the complexities and challenges for solving the nonlinear stochastic problem. The effect of nonlinearity on the problem was studied previously by several researchers. Bergan and Soreide [20] gave a brief review of some of the most important techniques used for solving nonlinear equations in structural problems. The mode superposition technique for performing nonlinear initial-boundary-value problems of structural dynamics was developed by Nickell [21]; also, Morris [22] studied the problem of cable bridge by using a similar method as well. Noor and Hartley [23] analyzed geometrically nonlinear laminated composite shells by using finite element method. Meanwhile, Wood and Zienkiewicz [24] studied geometrically nonlinear behaviors of elastic inplane oriented bodies by using a modified isoparametric finite element method. In general, we often use a total Lagrangian formulation to approach the problem of solid mechanics. Horrigmoe and Bergan [25] described a general formulation for geometrically nonlinear analysis of shells using flat finite elements, which was based on a updated Lagrangian description. Implicit time integration methods for solving nonlinear dynamic problems were discussed and evaluated by Bathe and Cimento [26]. Furthermore, Bathe and Bolourchi [27] extended the same analysis to geometric and material nonlinear plates and shells as well. Surana [28] studied statically geometric nonlinearity for a curved shell structure by using the total Lagrangian approach. Mei and Decha-Umphai [29] investigated the problem of a geometrically nonlinear beam for large amplitude free and forced vibrations by using finite element method. Recently, Chen et al. [30] dealt with the nonlinear frequency spectrum of a cantilever beam by using the iterative perturbation scheme. Chakravorty et al. [31] used an eight-noded isoparametric finite element to investigate free vibration behaviors of a linear doubly curved conoidal shell structure. Also, Ma et al. [32] studied statistical properties of a geometrically nonlinear mode spectrum for large amplitude forced vibration of beam structures. Shen et al. [33] adopted the singular perturbation technique to obtain the buckling load and postbuckling equilibrium paths of stiffened cylindrical shells under combined external pressure and axial compression. More recently, Cheng et al. [34] calculated geometrically nonlinear deformations of composite laminated plates using the perturbation finite element method. Lanzo [35] suggested a strategy for the imperfection sensitivity analysis of elastic thin-walled structures with notable residual stresses. This analysis was accomplished by means of a Koiter's perturbation approach. Wu et al. [36] obtained the three-dimensional solution of laminated conical shells subjected to axisymmetric loads by using the perturbation technique.

However, it is important and desirable to consider the uncertainty properties in shell structures. More recently, Kaminski [37] developed the generalized nth order perturbation technique in conjunction with finite element method to model one-dimensional linear elastostatics problems with one single random variable. In this study, the stochastic finite element method along with the perturbation technique was used to investigate the stochastic dynamic response of a geometrically nonlinear shell with structural randomness. In general, structural uncertainties might include the Young's modulus of elasticity, Possion's ratio, cross-section, length and geometric imperfections of shells. In this paper, only the Young's modulus of elasticity is considered as a stochastic field with respect to position, and then the stochastic finite element method along with perturbation method is used to investigate the stochastic dynamic response of a shell, which is loaded with a deterministic transverse dynamic load. In order to solve the nonlinearity of governing equations of shell structures, the well-known Newton–Raphson iteration procedure in conjunction with Newmark scheme is adopted to perform numerical analysis of the problem. Some statistical results from the present study are compared by those from the Monte Carlo simulation approach. Also, it should be noted that these statistical dynamic responses are quite useful for estimating the reliability of structures.

2. Formulation of the problem

2.1. Equations of motion

Consider the problem of a curved thin shell structure as shown in Fig. 1. A simple and efficient finite element named Serendipity eight-node element shown in Fig. 2 is adopted in this investigation. This kind of shell element contains five degrees of freedom at each of the eight nodes: u, v, w, α, β where the first three are translations in global directions and the last two are rotations about the local axes. The geometric layout of such elements is presented in Fig. 3, where the coordinates of any point can be expressed as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \sum_{i=1}^{8} \mathbf{N}_i \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \sum_{i=1}^{8} \mathbf{N}_i \zeta \frac{h_i}{2} \begin{bmatrix} l_{3i} \\ m_{3i} \\ n_{3i} \end{bmatrix}, \tag{1}$$