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Effect of strain hardening on the buckling of structural members and design codes recommendations

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Abstract

Some aspects of buckling behaviour of structural elements made of nonlinear materials with strain hardening, for which the stress–strain law has no yield plateau (stainless steels, aluminium alloys and others) are discussed. Considering compressed bars and plates made of materials with linear and nonlinear strain hardening, on the basis of tangent modulus theory and the deformation theory, we show that there exists a "threshold" value of the hardening factor below which the hardening does not influence the limit load. But if the hardening factor exceeds the threshold value then the hardening significantly increases the limit load and this increasing is not properly accounted for in some worldwide design codes.

Keywords: Buckling; Compressed bars; Plates; Nonlinear materials; Strain hardening; Design codes

1. Introduction

It is well known that the stress–strain relationships for many contemporary materials (in particular, stainless steels, high strength steels, aluminium alloys) are nonlinear ones with continuous hardening (without a yield plateau). In Fig. 1, the stress–strain curve for one of Russian stainless steels (X18H9T) is presented in dimensionless parameters (curve 1).

In the most design codes, such as Eurocode-3 [1], Polish and Russian codes [2,3] and others each material is characterized with "yield strength" f_y (or "basic yield strength" f_{yb}), and Perry–Robertson approach is used for buckling predictions. As a matter of fact, it means that instead of a real $\sigma-\varepsilon$ law the ideal elastic–plastic diagram of the material is assumed (curve 2 in Fig. 1), and no hardening is taken into account.

If the $\sigma-\varepsilon$ law includes a pronounced yield plateau, i.e., if there is the physical yield stress σ_y, then yield strength f_y has clear sense: it equals to σ_y (or (0.8–0.9)σ_y). In this case the replacement of the real $\sigma-\varepsilon$ curve with the ideal elastic–plastic law is natural and justified. But for materials without a sharp yield point and yield plateau the basic yield strength f_y has not a distinct physical sense. Usually it is assumed to be equal to 0.2% proof stress $\sigma_{0.2}$, but this choice, of course, is arbitrary.

The difficulty that we meet in nonlinear materials can be noted already in the case of the linear hardening (Fig. 2): should one take yield strength f_y equal to the proportionality limit or to $\sigma_{0.2}$?

The replacement of the real stress–strain relationships with the ideal elastic–plastic law is dictated by simplicity requirements. However, the question arises whether such a simplification is adequate for nonlinear materials without the yield plateau and whether it does not result in a systematic error in certain range of parameters of structural elements.

The influence of material nonlinearity on the buckling behaviour of metal structures was studied in a great amount of works, most of which are well known. Here we may refer, e.g., to review [4]. But in this paper we would like to emphasize some aspects of the problem, which did not draw enough attention in previous works. Considering compressed bars and plates made of materials with linear and nonlinear hardening, on the basis of the tangent modulus theory and the deformation theory, we show that in case of linear hardening there exists a "threshold" value

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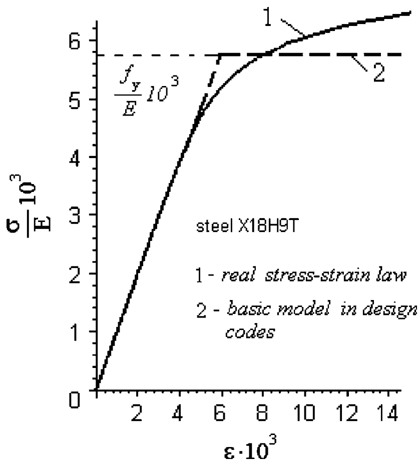

Fig. 1. The stress–strain law for stainless steel X18H9T and the idealized $\sigma-\varepsilon$ law.

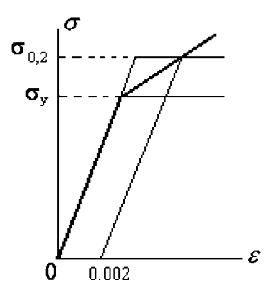

Fig. 2. The case of linear hardening.

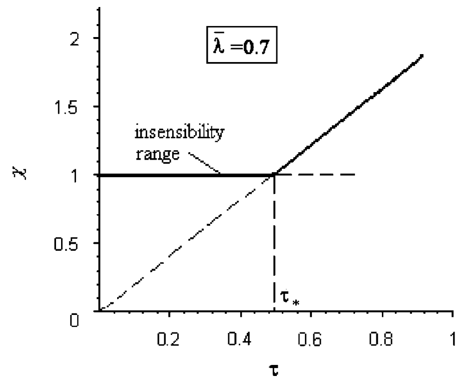

Fig. 3. The effect of hardening factor τ on the normalized critical stress for compressed bar.

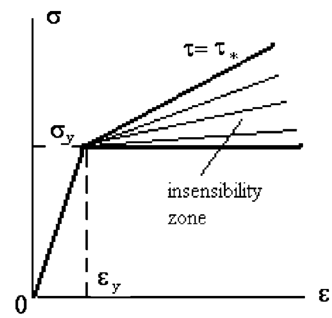

Fig. 4. Zone of insensibility of the critical stress to the hardening factor.

of the hardening factor. Below this value the hardening does not influence the limit load and so neglecting the hardening is justified. But if the hardening is beyond the threshold value then any choice of yield strength f_y results in a systematic error in certain slenderness range of the member.

2. Bars under compression

2.1. Linear hardening: ideal bars

Consider first an ideal bar made of material with linear hardening (a piece-linear stress–strain law)

$$\sigma = E\varepsilon_y + E_\tau(\varepsilon - \varepsilon_y) \quad (\sigma > \sigma_y), \tag{1}$$

where E_τ is the tangent modulus, σ_y is the proportionality limit, $\varepsilon_y = \sigma_y/E$. It is convenient to introduce dimensionless parameters, which have the order of 1:

$$\sigma^* = \frac{\sigma}{E}10^3, \quad \varepsilon^* = \varepsilon\,10^3. \tag{2}$$

In these parameters the law (1) takes the form ($\tau = E_\tau/E$ is the hardening factor)

$$\sigma^* = \varepsilon_y^* + \tau(\varepsilon^* - \varepsilon_y^*) \quad (\sigma^* > \sigma_y^*). \tag{3}$$

We consider here an ideal bar and so for buckling we may use the *tangent-modulus theory* (generally such an approach gives the low bound of theoretical predictions and satisfactorily correlates with experimental data). The critical stress is equal to

$$\sigma_{cr} = \tau\sigma_e, \tag{4}$$

where σ_e is the critical stress for elastic bar. Let us rewrite this formula, using the normalized stress and slenderness parameters adopted in design codes:

$$\chi = \frac{\sigma_{cr}}{\sigma_y}, \quad \bar{\lambda} = \frac{\lambda}{\lambda_1}, \tag{5}$$

where $\lambda_1 = \pi(E/\sigma_y)^{0.5}$. Then relationship (4) is reduced to

$$\chi = \frac{\tau}{\bar{\lambda}^2}. \tag{6}$$

Formula (6) holds only if $\tau/\bar{\lambda}^2 \geqslant 1$, otherwise $\sigma_{cr} = \min(\sigma_e, \sigma_y)$, so if $\tau/\bar{\lambda}^2 < 1$, then $\chi = 1$ for $\bar{\lambda} \leqslant 1$ and $\chi = 1/\bar{\lambda}^2$for $\bar{\lambda} > 1$. Hence function $\chi(\tau)$ for the case $\bar{\lambda} \leqslant 1$ has the form shown in Fig. 3 (for $\bar{\lambda} = 0.7$). If we consider $\sigma-\varepsilon$ laws with different values of the hardening factor τ, the critical stress remains constant and equals to σ_y in the τ range from 0 to a bound value $\tau_* = \bar{\lambda}^2$. It means that there exists an "insensibility zone" shown in Fig. 4; it is bounded with arrows $\tau = 0$ and $\tau = \tau_* = \bar{\lambda}^2$.

If $\sigma(\varepsilon)$ curve passes inside this zone, the critical stress equals to the yield stress σ_y (similarly to the ideal