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Stability problems of steel structures in the presence of stochastic and fuzzy uncertainty

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Abstract

General ideas and problems of probability approach and its utilization in the verification of structural design procedures of EUROCODES are mentioned. The paper is aimed at the probability study of the ultimate limit state of a steel compressed member designed economically according to EUROCODE 3. The theoretical failure probability (reliability index) vs. ratio of permanent to variable load action is calculated by means of the Monte Carlo simulation method. The misalignment of the failure probability according to EN1990 is analysed. Initial imperfections are generally considered as random variables and random fields. The non-linear beam FEM is used. The influence of initial curvature shape and size variability of the member axis on the variability of load-carrying capacity is investigated. The probabilistic analysis is supplemented with the fuzzy analysis of the influence of uncertainties on the failure probability.

Keywords: EUROCODE; Stability; Imperfections; Structural steel; Stochastic yield strength; Structural reliability; Fuzzy sets; Fuzzy reliability

1. Introduction

This paper deals with the probabilistic verification of reliability of steel bar structures designed according to the EUROCODE standard. Design procedures of EUROCODE standards, utilized in dimensioning of steel members, stems from the limit state methodology. The reliability of design is secured by partial safety factors.

Unified European standards EUROCODE ensure a satisfactory level of reliability provided that the required corresponding quality of production of metallurgical products in individual EU countries is met.

In the Czech Republic, material and geometric characteristics of steel products are controlled both by manufacturers and at independent scientific workplaces [1]. The greatest attention is paid to the monitoring and analysis of the random variable values of yield strength, material strength and ductility. It has been proved by means of comparison studies that statistical characteristics of yield strength, material strength and ductility of Czech and Austrian steel are in good concordance [2].

In stability problems, initial geometric imperfections of member cross-section and axis have a great influence on the load-carrying capacity of slender members under compression. With increasing member slenderness the influence of the variability of yield strength on the variability of load-carrying capacity decreases [3] and the influence of flexural rigidity EI (the product of Young's modulus E and the second moment of area I), which prevents buckling, increases [3]. The variability of load-carrying capacity of the bar is the most sensitive to the variability of initial imperfection of axial bar curvature with non-dimensional slenderness $\bar{\lambda} = 1.0$ [3].

The paper is aimed at the probabilistic analysis of the ultimate limit state of a compressed member of profile IPE 220 with $\bar{\lambda} = 1.0$. With aim at an accurate description of the influence of initial curvature of strut axis on the failure probability, size and shape imperfections of strut axis are modelled utilizing random fields [4] (see Fig. 1). Computer-based FEM modelling and simulation are required for the stochastic analysis.

The problem involves both aleatory and epistemic uncertainties. During structural design, an information on statistical characteristics of eventual loading is absent. Imprecision (fuzziness) of information on the random

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initial imperfections and their correlations presents a further source of uncertainty.

Newer mathematical approaches, which extend or depart from the probability theory, are available, e.g. in Refs. [5,6]. In order to obtain realistic results from stochastic inference, imprecision (fuzziness) of data has to be modelled quantitatively. This is possible by applying the fuzzy sets theory [7,8].

2. Initial curvature of the axis

In general, the axis of a real member is a curve; an ideally straight member is practically never concerned. Let us consider a long member with initial plane axial curvature (see Fig. 2) with unit weight g acting at the cross-sectional centroid, which is constant per unit length of curved axis. Next let the main central axis x_c of the curve about which the second moment of area of the curved axis of the member is minimal be sought.

An orthogonal coordinate system y_c vs. x_c is considered (see Fig. 2). The curve axis is divided into n equidistant members in the direction of axis x_c, resulting in a random sample of n members. Each member $i = 0, 1, \ldots, n$ is subjected to axial compressive loading passing through the first and the last node of the strut. The force line determines the local strut axis x_i, which is at angle α_i with respect to axis x_c.

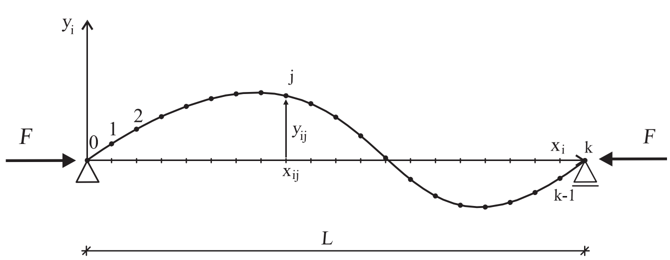

Fig. 1. Random field of strut axis curvature.

Each ith member is subdivided into k adjacent equidistant elements $k+1$ (see Fig. 1). The angle α_i of the local coordinate system y_i vs. x_i is dependent on the position of the initial y_{ci0} and final y_{cik} node of the member and may be determined from the relation

$$\tan(\alpha_i) = \frac{\Delta y_i}{\Delta x_i} = \frac{y_{cik} - y_{ci0}}{x_{cik} - x_{ci0}}. \tag{1}$$

Coordinates of the jth node in the coordinate system y_c vs. x_c are transformed into the local coordinate system y_i vs. x_i according to the relations

$$y_{ij} = (y_{cij} - y_{ci0})\cos(\alpha_i) - x_{cij}\sin(\alpha_i), \tag{2}$$

$$x_{ij} = (y_{cij} - y_{ci0})\sin(\alpha_i) + x_{cij}\cos(\alpha_i). \tag{3}$$

For $\Delta y_i \ll \Delta x_i$ and $(y_{cij} - y_{ci0}) \ll 1$ it holds approximately that $x_{ij} \approx x_{ij}$, $x_{ij} \approx x_{cij}$, which is a frequent case practically.

The initial deformation of the jth node in the coordinate system y_c vs. x_c is a random variable, which will be denoted as y_{cj}. When the number of struts n is a sufficiently large number it holds that the mean value m_{ycj} of random variable y_{cj} of the jth node is approximately zero:

$$m_{ycj} = \frac{1}{n}\sum_{i=1}^{n} y_{cij} \approx 0 \quad \text{for } j = 0, 1, \ldots, k. \tag{4}$$

The shape of initial curvature of the strut axis depicted in Fig. 1 represents the ith random observation from n struts. Let us denote the random deviation of the jth node in the local coordinate system y vs. x as y_j (see Fig. 3). It can be illustrated (e.g. by Monte Carlo simulation) that the mean value m_{yj} of random deviation y_j is equal to zero:

$$m_{yj} = \frac{1}{n}\sum_{i=1}^{n} y_{ij} \approx 0 \quad \text{for } j = 0, 1, \ldots, k, \tag{5}$$

where y_{ij} was evaluated according to Eq. (2). The shape of the random curvature is given by the correlation of variables y_j amongst $k-1$ nodes with $y_0 = y_k = 0$. The correlations amongst variables y_{cj} are approximately the

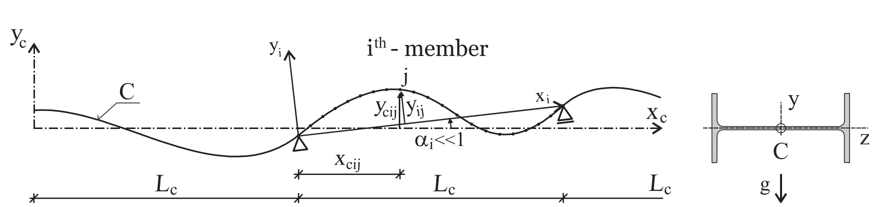

Fig. 2. Global and local coordinate system of axial curvature.

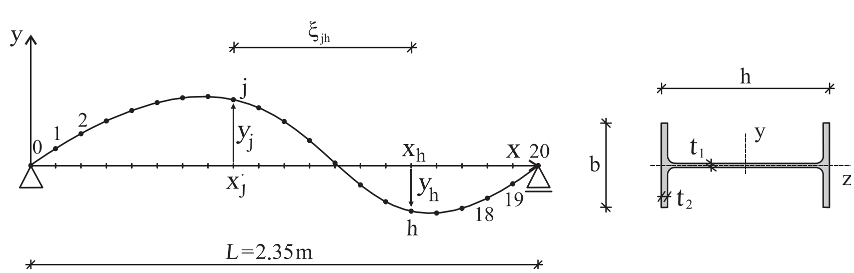

Fig. 3. Geometrical imperfections of strut axis curvature and cross-section dimensions.