Available online at www.sciencedirect.com
ScienceDirect
Thin-Walled Structures 45 (2007) 888–892
THIN-WALLED STRUCTURES
www.elsevier.com/locate/tws

Criteria of dynamic buckling estimation of thin-walled structures

T. Kubiak*

Department of Strength of Materials and Structures (K-12), Technical University of Łódź, Stefanowskiego 1/15, 90-924 Łódź, Poland

Available online 17 October 2007

Abstract

The paper deals with dynamic buckling of thin-walled structures (plates and beam-columns with open cross-section) subjected to compressive rectangular pulse loading. The local, global and interactive dynamic buckling was analysed. Author proposes the new criterion for critical amplitude of pulse loading leading to stability loss. The proposed criterion is a modification of quasi-bifurcation criterion formulated by Kleiber, Kotula and Saran. Results obtained using proposed criterion were compared with other well-known criteria (Volmir (V) and Budiansky–Hutchinson (B–H) criterion).

Keywords: Dynamic buckling; Thin-walled structures; Dynamic buckling criteria

1. Introduction

Dynamic buckling of thin-walled structures subjected to pulse compressive loading and the critical amplitude of pulse loading leading to stability loss was considered. The dynamic buckling occurs when the loading process is of intermediate amplitude and the pulse duration is close to the period of fundamental natural flexural vibrations (in range of milliseconds). In such case the effects of dumping can be neglected [1]. It should be noted that dynamic stability loss may occur only for structures with initial geometric imperfections, therefore the dynamic bifurcation load does not exist. For the ideal structures (without any geometrical imperfection) the critical buckling amplitude of pulse loading tends to infinity [2]. The dynamic buckling load should be defined on the basis of the assumed buckling criterion.

Dynamic buckling of thin-walled structures has been discussed in many works for more than 50 years [1–11]. In the majority of studies numerous simplifications have been made to allow in practice for an effective analysis of stability of the thin-walled structure. Mathematical models tend to aim at higher precision and closer approximation of real structures, which enables one to analyse more and more exactly the phenomena occurring during and after the loss of dynamic stability. The precise mathematical criteria were formulated for structures having unstable postcritical equilibrium path or having limit point [2,10]. But for the structures having stable postbuckling equilibrium path (thin plate, thin-walled beam-columns with minimal critical load corresponding to local buckling) the precise mathematical criterion was not defined.

In world literature a lot of criteria can be found. In the 1960s of the 20th century, Volmir (V) [11] proposed a criterion for plates subjected to in-plane pulse loading. Next criteria were formulated by Budiansky and Hutchinson (B–H) [2,4,6]. In 1987, Kleiber et al. [12] analysed dynamic behaviour of rod structures subjected to pulse loading (Heaviside's function) and formulated quasi-bifurcation criterion that allows to find critical amplitude of pulse loading. In the end of 1990s Ari-Gur and Simonetta [3] formulated four criteria—two of them are collapse-type buckling criteria. The failure criterion was proposed by Petry and Fahlbush [9], who suggest that for structures with stable postbuckling equilibrium path the B–H criterion is very restrict because it does not take into account load carrying capacity of the structure.

Till now the most popular and easy to apply are criteria defined by Volmir and B–H. Volmir [11] analysed a thin plate subjected to in-plane compressive loading with different pulse shape, and defined the following criterion:

> Dynamic critical load corresponds to the amplitude of pulse load (of constant duration) at which the maximum

*Tel.: +484263212221; fax: +48426364985.
E-mail address: tomasz.kubiak@p.lodz.pl

0263-8231/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.tws.2007.08.039

plate deflection is equal to some constant value k (k-half or one plate thickness).

In many publications, the dynamic buckling load is determined on the basis of stability criterion by B–H [2,4,6]. However, this criterion was formulated for shell structures but it also can be used for the plate structures [5,7–9]. B–H noticed that in some range of the amplitude value, the deflection of structures grows more rapidly than in other. B–H formulated the following criterion:

> Dynamic stability loss occurs when the maximum deflection grows rapidly with the small variation of the load amplitude.

To find the critical value defining the dynamic buckling first of all the dynamic response should be analysed. The problem is investigated on the basis of the asymptotic analytical–numerical method [13]. In order to obtain the equations of plate, the non-linear theory of orthotropic thin-walled plates has been modified in such a way that it additionally accounts for all components of inertia forces. The differential equations of motion have been obtained from Hamilton's Principle, taking into account Lagrange's description, full Green's strain tensor for thin plates and Kirchhoff's stress tensor.

2. Solution method

The problem was solved using the proposed analytical–numerical method [8] which allows to analyse the static buckling, post buckling behaviour and dynamic response for the thin-walled structure composed of plates made of isotropic or orthotropic material.

The thin-walled prismatic columns of the length l composed of rectangular plate segments interconnected along longitudinal edges are considered. Analysed columns were simply supported at loaded ends. The plates are rectangular and can be isotropic or orthotropic with the principal axes of orthotropy parallel to the plate edges [8,14]. The material all the plates are made of is subjected to Hooke's law. It is assumed that the loaded edges remain straight and parallel during loading. Additionally, it is assumed that normal and shear forces disappear along the unloaded edges.

For the ith plate component, the precise geometrical relationships are assumed in order to enable the consideration of both out-of-plane and in-plane bending of each plate [8,14]:

$$\begin{aligned}
\varepsilon_{ix} &= u_{i,x} + \tfrac{1}{2}(w_{i,x}^2 + u_{i,x}^2 + v_{i,x}^2),\\
\varepsilon_{iy} &= v_{i,y} + \tfrac{1}{2}(w_{i,y}^2 + u_{i,y}^2 + v_{i,y}^2),\\
2\varepsilon_{ixy} &= \gamma_{ixy} = u_{i,y} + v_{i,x} + w_{i,x}w_{i,y}\\
&\quad + u_{i,x}u_{i,y} + v_{i,x}v_{i,y},
\end{aligned} \tag{1}$$

where u_i, v_i, w_i are displacement components of the middle surface of the ith plate in the x_i, y_i, and z_i directions, correspondingly.

The differential equations of equilibrium obtained from Hamilton's Principle for a single plate can be written as

$$\begin{aligned}
&-\rho_i h_i \ddot{u}_{i,x} + N_{xi,x} + N_{xyi,y} + (N_{yi}u_{i,y})_{,y}\\
&+ (N_{xi}u_{i,x})_{,x} + (N_{xyi}u_{i,x})\\
&(N_{xyi}u_{i,y})_{,x} = 0,\\
&-\rho_i h_i \ddot{v}_i + N_{xyi,x} + N_{yi,y} + (N_{xi}v_{i,x})_{,x}\\
&+ (N_{yi}v_{i,y})_{,y} + (N_{xyi}v_{i,x})_{,y}\\
&+ (N_{xyi}v_{i,y})_{,x} = 0,\\
&-\rho_i h_i \ddot{w}_i + (N_{xi,x} + N_{xyi,y})w_{i,x}\\
&+ (N_{yi,y} + N_{xyi,x})w_{i,y} + N_{xi}w_{i,xx}\\
&+ N_{yi}w_{i,yy} + 2N_{xyi}w_{i,xy} + M_{xi,xx}\\
&+ 2M_{xyi,xy} + M_{yi,yy} = 0.
\end{aligned} \tag{2}$$

Let us obtain the equations of motion of a compressed plate assuming that the natural modes of vibration coincide with the buckling modes (it is the case, in particular, for simply supported plates). Let λ is a load factor, $\mathbf{U}_i$ ($i=1,N$) are the linear buckling modes with the critical load factor values λ_i close to the minimal critical value $\lambda_{\min}$. We assume the following expansion of dynamic displacements field (Koiter's type expansion for the buckling problem—see [13,14]):

$$\mathbf{U} \equiv (u, v, w) = \lambda\mathbf{U}_0 + \xi_i(t)\mathbf{U}_i + \xi_i(t)\xi_j(t)\mathbf{U}_{ij} + \dots, \tag{3}$$

where $\xi_i = w_i/h_1$ is the amplitude of ith mode (normalized, in given case, by the condition of equality of the maximal deflection to the thickness of the first component plate h_1), $\mathbf{U}_{ij}$ are the second order displacement fields, summation is supposed on repeated indexes.

The static part (inertia forces were neglected) of the system of the ordinary differential equilibrium equations (2), the first and second order approximations in the xy plane was solved by the modified transition matrix method. The state vector at the final edge based on the state vector at the initial edge was found by numerical integration of the differential equations (2) in transverse direction using the Runge–Kutta formulae by means of the Godunov orthogonalization method [14,15]. The above method allows to find non-linear postbuckling coefficients: a_0, a_s, a_{jks}, a_{jkls} performed in equation describing postbuckling equilibrium path [14]. For the structure contains geometric imperfections $\bar{\mathbf{U}}$ (only linear initial imperfections determined by the shape of ith buckling modes), where $\bar{\mathbf{U}} = \xi_i^* \mathbf{U}_i$ then, similarly to the Koiter's theory for the buckling problem [13,14] the potential energy corresponding to the equilibrium path can be written as follows:

$$P = \frac{1}{2}a_0\lambda^2 + \frac{1}{2}\sum_s a_s \xi_s^2(t)\left(1 - \frac{\lambda}{\lambda_s}\right) + \frac{1}{3}a_{ijs}\xi_i(t)\xi_j(t)\xi_k(t)$$