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The dynamic stability of a laboratory model of a truck crane

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Abstract

In the work the dynamic stability of laboratory model of the truck crane is considered. The results in the form of frequency curves for changing the geometry of the system have been presented. Solution of the Mathieu equation enables one to determine the dynamic stability regions of the system. It has been found that, for each of the studied examples, there exists such a rope length for which the critical value of the coefficient a in the Mathieu equation is obtained. That means that for specified geometrical and load conditions, the system may loose its dynamic stability (unless vibration damping is considered).

Keywords: Truck crane; Dynamic instability; Mathieu equation

1. Introduction

A truck crane is one of the most complicated machines in respect of dynamics. There are many works dealing with research into the dynamics of truck cranes and their telescopic booms. In 2005, a new monograph on the modelling of and research into the dynamics of a self-propelled truck crane [1] was published. A model of the truck crane, which takes into account all possible control for the real system operation, is presented in [2]. In [3] analysis of free and parametrical vibrations of the system of changes in crane radius of a DUT 0203 crane were carried out, while in [4]—an analysis of the free vibrations of a laboratory model of truck crane is conducted. The influence of flexible soil foundation on the dynamic stability of a crane boom during its rotation is considered in [5]. Research into the stability of a truck crane during the control of different operational motions is performed in [6]. In [7], based on flexible multibody theory, the complete dynamic simulation for crane rotating is studied. In [8], the dynamic analysis of the truck crane model during an arbitrary sequence of operational motions, such as: lifting, lowering, rotation or changes in the crane radius is presented.

The considerations, presented in this paper, take into account analysis of the dynamic stability of a laboratory model of a truck crane. The vibrations of the system in the boom's lifting plane are analyzed. The possible existence of parametric resonance in the system is investigated. It can be stated that at a determined geometry of the system (the length of the boom and its inclination angle) there are such rope lengths for which the critical value of coefficient a in the Mathieu equation is obtained. This means that a system fulfilling determined geometrical and load conditions (without taking into consideration the damping) may lose dynamic stability.

2. Physical model of the system

The stand [9], for testing the dynamics of a truck crane, is a laboratory model of a real truck crane made on a scale 1:5. All operational motions of the model are controlled by using of hydraulic systems.

The following features were taken into account in the physical model of the tested system:

(a) the reacting force of the load hanging on a rope, lifted or stopped during lowering, on the tested system (the force in the form $P(t) = P_0 + S\cos \nu t$),
(b) actual geometry of the system defined by α and δ angles,
(c) equivalent masses (M_{zw}, M_{r1} and M_{r2}) at the support point of boom by the hydraulic cylinder and at the support point of the basic element of the boom

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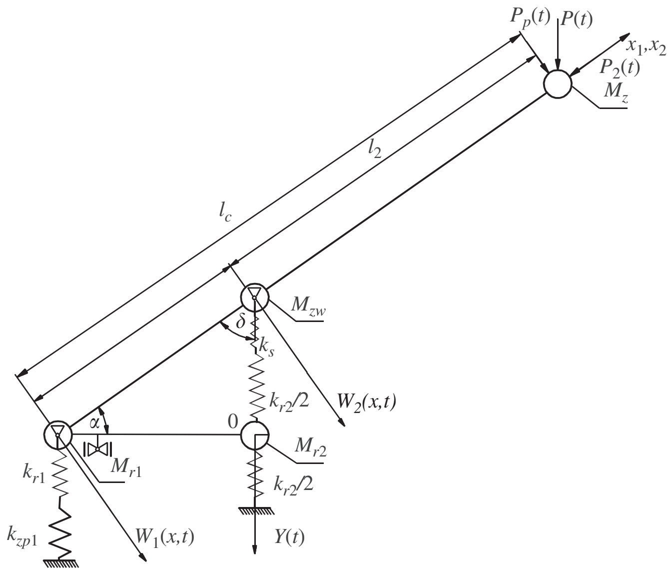

Fig. 1. The physical model of the tested system.

(considering: mass of the boom, mass of the rotating frame, mass of the piston rod of the supported hydraulic cylinder, mass of counterweight). The mass of the head of boom M_z has been also considered.
(d) equivalent rigidity of the boom,
(e) elasticity: of hydraulic cylinder for changes in the crane radius k_s, of rotating platform k_{r1}, of model of chassis frame k_{zp1}.

The physical model of the tested system is presented in Fig. 1.

During formulation of the mathematical model the following simplifications were assumed:

(a) beams of the system are Bernoulli–Euler's beams,
(b) the model of the chassis frame of truck crane does not consider its mass,
(c) a spring modelling the rigidity of chassis frame is applied at the fixing point of the boom in rotating frame.

3. Mathematical model

The mathematical model is formulated by means of Hamilton's principle [10] and as a result the equations of motion and natural boundary conditions are obtained. The equations of motion take the following form:

$$E_iJ_i\frac{\partial^4 W_i(x_i,t)}{\partial x_i^4}+P_i(t)\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}+\rho_iA_i\frac{\partial^2 W_i(x_i,t)}{\partial t^2}=0,\quad i=1,2, \tag{1}$$

where E_iJ_i the flexural rigidity of ith beam, ρ_i the material density of ith beam, A_i the cross-section of ith beam, $P_i(t)=P_{i0}+S\cos vt$.

$$M_{r2}\frac{\partial^2 Y(t)}{\partial t^2}+\frac{k_{r2}}{2}Y(t)-k_{z1}(W_2(0,t)\cos\alpha-Y(t))=0, \tag{2}$$

where, $k_{z1}=k_s(k_{r2}/2)/k_s+(k_{r2}/2)$,

while the boundary conditions (at $S=0$) are

$$E_1J_1W^{III}(0,t)-k_{r1}W_1(0,t)\cos\alpha-M_{r1}\ddot{W}_1(0,t)=0,\quad W_1^{II}(0,t)=0, \tag{3,4}$$

$$W_1(l_1,t)=W_2(0,t),\quad W_1^{I}(l_1,t)=W_2^{I}(0,t),\quad W_2^{II}(l_2,t)=0, \tag{5-7}$$

$$E_1J_1W_1^{II}(l_1,t)=E_2J_2W_2^{II}(0,t),\quad E_2J_2W_2^{III}(l_2,t)+P_{20}W_2^{I}(l_2,t)-M_z\ddot{W}_2(l_2,t)=-P_{p0}, \tag{8,9}$$

$$E_1J_1W_1^{III}(l_1,t)+P_{10}W_1^{I}(l_1,t)-E_2J_2W_2^{III}(0,t)-k_{z1}(W_2(0,t)\cos\alpha-Y(t))-M_{zw}\ddot{W}_2(0,t)=0 \tag{10}$$

Substitution of

$$W_i(x_i,t)=w_i(x_i)e^{j\omega t} \tag{11}$$

and

$$Y(t)=ye^{j\omega t} \tag{12}$$

into Eqs. (1)–(2) and into conditions (3,4)–(10) leads to (at $S=0$)

$$E_iJ_iw_i^{IV}(x_i)+P_{i0}w_i^{II}(x_i)-\rho_iA_i\omega^2w_i(x_i)=0 \tag{13}$$

and

$$-M_{r2}\omega^2+\frac{k_{r2}}{2}y-k_{z1}(w_2(0)\cos\alpha-y)=0, \tag{14}$$

together with the boundary conditions

$$E_1J_1w_1^{III}(0)-k_{r1}w_1(0)\cos\alpha+M_{r1}\omega^2w_1(0)=0,\quad w_1^{II}(0)=0, \tag{15,16}$$

$$w_1(l_1)=w_2(0),\quad w_1^{I}(l_1)=w_2^{I}(0),\quad w_2^{II}(l_2)=0, \tag{17-19}$$

$$E_1J_1w_1^{II}(l_1)=E_2J_2w_2^{II}(0),\quad E_2J_2w_2^{III}(l_2)+P_{20}w_2^{I}(l_2)+M_z\omega^2w_2(l_2)=-P_{p0}, \tag{20,21}$$

$$E_1J_1w_1^{III}(l_1)+P_{10}w_1^{I}(l_1)-E_2J_2w_2^{III}(0)-k_{z1}(w_2(0)\cos\alpha-y)+M_z\omega^2w_2(0)=0 \tag{22}$$

Constant y, present in condition (22), was determined from Eq. (14).

The solution of Eq. (13) is the following function:

$$w_i(x_i)=C_1\sinh(\lambda_ix_i)+C_2\cosh(\lambda_ix_i)+C_3\sin(\bar{\lambda}_ix_i)+C_4\cos(\bar{\lambda}_ix_i), \tag{23}$$

where, C_1–C_4—constants, $\lambda_i=\sqrt{-(\beta_i^2/2)+\sqrt{(\beta_i^4/4)+\gamma_i}}$, $\bar{\lambda}_i=\sqrt{\beta_i^2/2+\sqrt{\beta_i^4/4+\gamma_i}}$, where, $\beta_i^2=P_{i0}/E_iJ_i$, $\gamma_i=\rho_iA_i\omega^2/E_iJ_i$.

The solution of the boundary condition leads to a homogenous system of eight equations in relation to unknown constants C_n $(n=1,2,3,4)$. The above system