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Axi-symmetrical deflection and buckling of circular porous-cellular plate

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Abstract

The main goal of this paper is a solution of the problem of buckling and deflection. A circular porous plate with simply supported edge under radial uniform compression and uniformly distributed load (pressure) is considered. Mechanical properties of the isotropic porous material vary across the thickness of the plate. Middle plane of the plate is its symmetry plane. A field of displacements (geometric model of nonlinear hypothesis) is described. The principle of stationarity of the total potential energy allowed to get a system of differential equations that govern the plate stability. A critical load and a deflection are determined. The results obtained for porous plates are compared to homogeneous circular plates.

Keywords: Circular plate; Elastic buckling; Porous-cellular metal

1. Introduction

Problem of deflection and buckling of the plates is described in many works and monographs. Some of them deal with the classical (Kirchhoff) theory which is not adequate in providing accurate buckling. This is due to the effect of transverse shear strains. Shear deformation theories provide accurate solutions compared to the classical theory. During the last several years this problem has been developed by many authors. The simplest, widely used approach for modelling plates made of non-homogeneous material is using Kirchhoff–Love'a hypothesis for describing the displacement field and taking into account modified forms of stiffness coefficients. For example, this way of modelling was used by Ambartsumian [1] in his monographs. The first shear deformation theory is presented by Vinson [2]. Later, many hypotheses, which include shearing, have been formulated. One of the monograph devoted to this problem is the work of Wang et al. [3], where authors presented not only their own solutions but also a review of previous attempts to model beams and plates. A comparison of theories used for modelling compressed and bent multilayered composite plates is presented in Chattopadhyay [4]. Banhart [5] provided a comprehensive description of various manufacturing processes of metal foams and porous metallic structures. Structural and functional applications to various industrial sectors are discussed. Instead, porous plates and beams with varying properties were described by Malinowski and Magnucki [6], Magnucki and Stasiewicz [7,8] and Magnucki [9] where also a nonlinear hypothesis was assumed. Porous-cellular materials exist in the nature, for example, structure of bones cross-section. Taking into account these structures, many searchers describe and manufacture similar structures. Suresh and Mortensen [10] presented detailed technology of these graded materials and its mechanical properties.

Presented circular porous plate is a generalization of sandwich structure. These circular plates, for example as flat baffle plate are used in water filter or horizontal cylindrical pressure vessels. Flat baffle plates of vessels are loaded by pressure and radial compression. So this paper is concerned with the problem of deflection and buckling of the plate.

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2. Displacements of a porous plate

This work is divided into two parts. Both of them are concerned with a circular porous plate with simply supported edge. But in the first one, the plate under uniformly distributed load is described and in the second one, the plate under radial uniform compression. Mechanical properties of the material vary through the thickness of the plate. Minimal value of Young's modulus occurs in the middle surface of the plate and maximal values at its top and bottom surfaces. For such a case, the Euler–Bernoulli or Timoshenko plate theories do not correctly determine displacements of the plate's cross-section. Wang [3] discussed in details the effect of non-dilatational strain of middle layers on bending of plates subject to various load cases. A porous plate (Fig. 1) is a generalized sandwich plate. The material is of continuous mechanical properties. The top and the bottom plate surfaces are made of non-porous material, while maximal porosity of the material occurs in the middle surfaces of the plate. The degree of porosity varies in normal direction. This plate is described in polar (cylindrical) coordinate system with the z-axis in the normal direction. The moduli of elasticities and mass density vary continuously too, as follows:

$$E(z) = E_1[1 - e_0\cos(\pi\zeta)], \quad G(z) = G_1[1 - e_0\cos(\pi\zeta)],$$
$$\varrho(z) = \varrho_1[1 - e_m\cos(\pi\zeta)], \tag{1}$$

where, e_0 is the coefficient of plate porosity, $e_0 = 1 - E_0/E_1$, E_0, E_1 the Young's moduli at $z = 0$ and $z = \pm h/2$, respectively, G_0, G_1 the shear moduli for $z = 0$ and $z = \pm h/2$, respectively, G_j the relationship between the moduli of elasticy for $j = 0, 1$, $G_j = E_j/[2(1+v)]$, v the Poisson's ratio (constant for the entire plate), e_m the dimensionless parameter of mass density, $e_m = 1 - \varrho_0/\varrho_1$, ϱ_0, ϱ_1 the mass density for $z = 0$ and $z = \pm h/2$, respectively, ζ the dimensionless coordinate, $\zeta = z/h$, h the thickness of the plate.

Choi and Lakes [11] presented mechanical properties for porous materials. Taking into account the results of investigations of this paper the relation between dimensionless parameter of mass density $e_m = 1 - \varrho_0/\varrho_1$ and dimensionless parameter of the porosity of the metal foam e_0 is defined as follows: $e_m = 1 - \sqrt{1 - e_0}$.

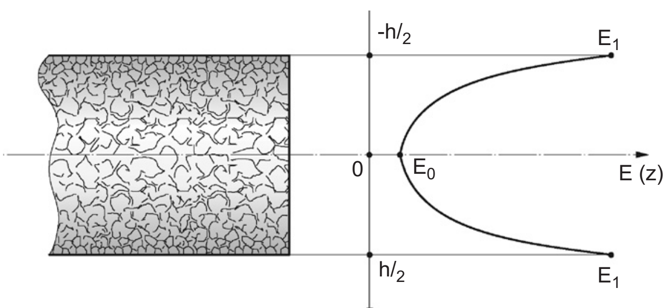

Fig. 1. Scheme of porous plate.

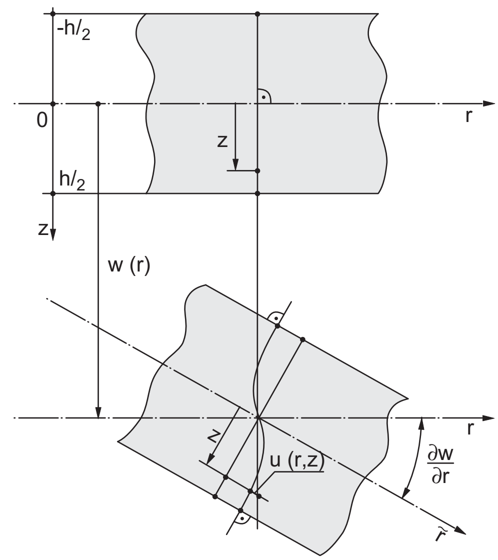

Fig. 2. Scheme of a deformation of a plane cross-section of the beam—the nonlinear hypothesis.

The physical model of deformation of a plane cross-section of the plate (the nonlinear hypothesis) is shown in Fig. 2. The cross-section, being initially planar surface, becomes curved after the deformation. The surface perpendicularly intersects the top and the bottom surfaces of the plate. This geometric model is analogous to the broken-line hypothesis applied to three-layered structures. A field of displacements in any cross-section is assumed in the following form:

$$u(r,z) = -h\left\{\zeta\frac{\mathrm{d}w}{\mathrm{d}r} - \frac{1}{\pi}[\psi_1(r)\sin(\pi\zeta) + \psi_2(r)\sin(2\pi\zeta)\cos^2(\pi\zeta)]\right\},$$

$$w(r,z) = w(r,0) = w(r), \tag{2}$$

where $u(r,z)$ is the longitudinal displacement along the r-axis, $w(r)$ the deflection (displacement along the z-axis), $\psi_1(r)$, $\psi_2(r)$ the dimensionless functions of displacements.

The strains are linear and components of the strain field describing the geometric relationships are defined as follows:

$$\varepsilon_r = \frac{\partial u}{\partial r} = -h\left\{\zeta\frac{\mathrm{d}^2w}{\mathrm{d}r^2} - \frac{1}{\pi}\left[\frac{\mathrm{d}\psi_1}{\mathrm{d}r}\sin(\pi\zeta) + \frac{\mathrm{d}\psi_2}{\mathrm{d}r}\sin(2\pi\zeta)\cos^2(\pi\zeta)\right]\right\},$$

$$\varepsilon_\varphi = \frac{u}{r} = -h\left\{\frac{1}{r}\zeta\frac{\mathrm{d}w}{\mathrm{d}r} - \frac{1}{\pi}\left[\frac{1}{r}\psi_1\sin(\pi\zeta) + \frac{1}{r}\psi_2\sin(2\pi\zeta)\cos^2(\pi\zeta)\right]\right\},$$