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Closed-form analysis of the buckling loads of uniaxially loaded blade-stringer-stiffened composite plates considering periodic boundary conditions

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Abstract

A closed-form solution for the buckling loads of compressively loaded composite plates longitudinally stiffened by blade stringers is developed. The approach is based on adequate formulations for the buckling shapes for both the plate and the stringers in the form of rather simple functional representations. As a by-product, a closed-form solution for a clamped stringer plate is derived. Furthermore, a full series expansion is developed for comparison purposes. The method works with satisfying accuracy and requires negligible computational expenses when compared to accompanying finite element calculations.

Keywords: Buckling; Composites; Stiffened plates; Blade stringers

1. Introduction

Composite materials have found an ever-increasing use in many branches of engineering where lightweight aspects are crucial factors. Classical fields of application are aeronautical and astronautical engineering. Since many structural elements in aerospace structures are of a rather thin-walled nature, the assessment of their buckling behaviour is a predominant aspect of analysis and design.

In many cases, thin-walled and plate-like composite structures are braced by stiffening elements (often also referred to as stringers), the application of which generally is very beneficial concerning the stability behaviour and which leads to a significant increase in the resultant buckling loads. One very common type of stringer is the so-called blade stringer which actually is a plate perpendicularly attached to the composite plate. A typical arrangement of this class can be found in Fig. 1, left portion, where a composite plate stiffened by several blade stringers is depicted. The plate width between two stringers is denoted as b (often referred to as stringer pitch), the stringer height is h. The plate has the length a and is subjected to some inplane normal loading. When considering the buckling behaviour of such a stiffened plate, there are several possibilities into which kind of shape the plate will buckle. Besides some global buckling mode in which the plate as well as the stringers will have to endure out-of-plane displacements, there are several possibilities for local buckling modes (Fig. 2) that are likely to occur, provided that the stringers are of a sufficient bending stiffness such that they remain immovable in the state of the onset of buckling. Firstly, for many application cases it can be assumed that some kind of mixed buckling mode will occur (Fig. 2, upper portion) characterized by a buckling of the plate in combination with a simultaneous buckling mode of the blade stringers. Secondly, a limit case can be found for very stiff stringers which will enforce a buckling of the plate such that the buckling shape corresponds to a clamped plate (Fig. 2, middle portion, denoted as limit case I). Thirdly, the second limit case is a sufficiently strong plate such that only the stringers will develop a buckling mode while the plate remains immovable (Fig. 2, lower portion, denoted as limit case II) which corresponds to the case of a rigidly clamped stringer. As Fig. 2 reveals, the resultant buckling shapes obviously exhibit periodicity properties in

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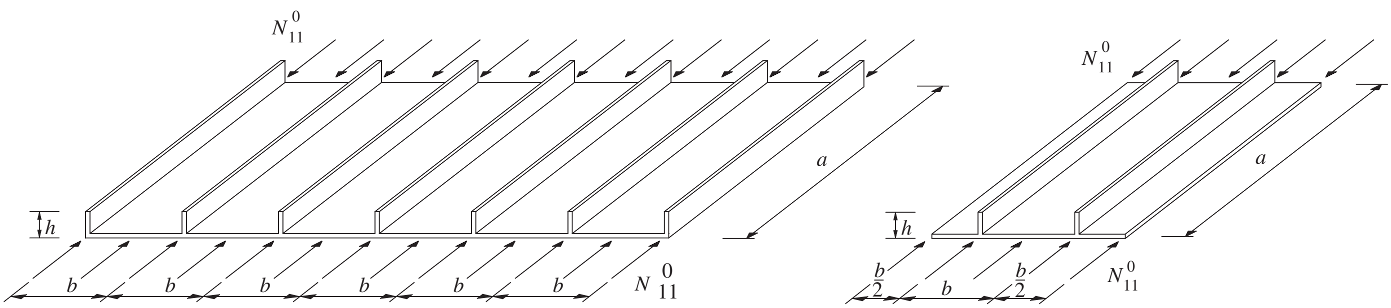

Fig. 1. Compressively loaded composite plate stiffened by blade stringers (left portion), representative unit cell for local buckling analysis (right portion).

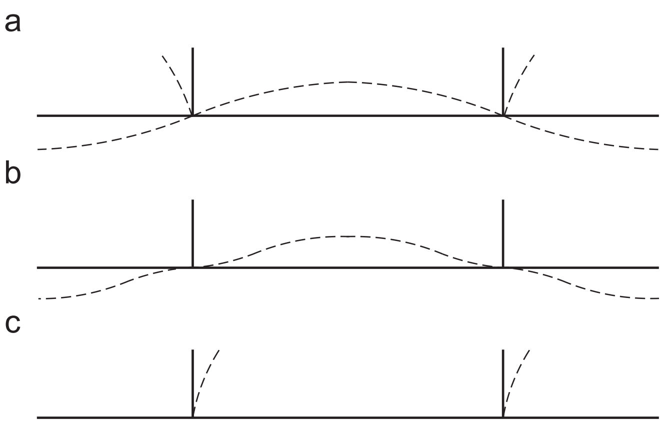

Fig. 2. Basic local buckling modes, (a) combined plate and stringer buckling, (b) limit case I: plate buckling in the case of stiff stringers, (c) limit case II: stringer buckling in the case of stiff plate.

such a way that it is possible to reduce the analysis of a rather complex structural situation of a composite plate with multiple stringers (Fig. 1, left portion) to the analysis of a representative unit cell (Fig. 1, right portion) without loss of generality, provided that we are dealing with immovable stringers at all locations and that the plate and stringer properties as well as the loading conditions do not vary with respect to the width coordinate. Of course, the employed analysis approach must be chosen such that the mentioned periodicity properties are captured adequately.

The buckling and postbuckling behaviour of stiffened isotropic and composite plates and shells has been the topic of an uncountable number of investigations throughout the last decades. Since a detailed survey cannot be performed in the framework of this contribution, in the following only some very recent works are shortly mentioned. Mallela and Upadhyay [1] discussed the buckling behaviour of simply supported blade-stringer-stiffened composite panels under inplane shear and performed detailed parameter studies using the finite element method. Möcker and Reimerdes [2] investigated the buckling and postbuckling characteristics of curved stiffened composite panels by using the finite strip method. Petrisic et al. [3] considered simply supported square plates with strengthening or weakening bands under inplane normal and shear loads and reported extensive parameter studies which were performed using the finite element method. Buermann et al. [4] introduced an efficient semi-analytical model for the analysis of the local postbuckling behaviour of isotropic cylindrical panels stiffened by stringers and frames. Therein, the description of the buckling shapes of the plate and the stiffeners was done by using a Fourier approximation. Zimmermann et al. [5] conducted buckling and postbuckling experiments and computations on axially compressed stiffened CFRP curved panels. Linde et al. [6] described a numerical model for the parametric modelling and numerical simulation of test shells and presented results for welded and fibre metal laminate panels.

In this contribution, a closed-form analysis method for the buckling loads of blade-stringer stiffened orthotropic composite plates under uniaxial compression is developed. The approach depends on explicit functional formulations for the buckling shapes of the plate as well as for the stringers accounting for periodicity properties resulting from the given structural situation and as such follows the method of Buermann et al. [4]. The approach allows for the assessment of combined buckling modes such as given in Fig. 2, upper portion, as well as for local buckling modes of the plate (Fig. 2, middle portion) and the stringers (Fig. 2, lower portion) in a straightforward manner. Furthermore, the methodology leads to an explicit closed-form analytical novel expression for the buckling load of a clamped stringer. The results by the closed-form solution for the buckling of the stiffened panel are compared with a full series expansion on the one hand and to accompanying finite element calculations on the other hand. In both the cases, a very satisfying agreement is found which renders the presented method a very trustworthy approach while requiring negligible computational expenses.

2. Analysis settings

2.1. Structural situation

Let us consider a representative unit cell of a stiffened composite plate as depicted in Fig. 3, left portion. The unit cell is braced by two blade stringers. The plate field between the stringers has the width b while both outer plate fields have the width $b/2$. The plate is subjected to a uniaxial compressive load N_{11}^0, and the stringers are loaded