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Generalised beam theory to analyse the buckling behaviour of circular cylindrical shells and tubes

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Abstract

A formulation of generalised beam theory (GBT) developed to analyse the elastic buckling behaviour of circular hollow section (CHS) members (cylinders and tubes) is presented in this paper. The main concepts involved in the available GBT are adapted to account for the specific aspects related to cross-section geometry. Taking into consideration the kinematic relations used in the theory of thin shells, the variation of the strain energy is evaluated and the terms are physically interpreted, i.e., they are associated with the geometric properties of the CHS. Besides the set of shell-type deformation modes, the formulation also includes axisymmetric and torsion deformation modes. In order to illustrate the application and capabilities of the formulated GBT, the local and global buckling behaviour of CHS members subjected to (i) compression (columns), (ii) bending (beams), (iii) compression and bending (beam-columns) and (iv) torsion (shafts), is analysed. Moreover, the GBT results are compared with estimates obtained by means of shell finite element analyses and are thoroughly discussed.

Keywords: Cylinders; Tubes; Generalised beam theory (GBT); Circular hollow sections (CHS); Local buckling; Compression; Bending; Torsion

1. Introduction

The stability of circular hollow section (CHS) members (cylinders and tubes) has been studied both analytically and experimentally since the beginning of 20th century. The buckling of axially compressed cylindrical members was first approximately analysed by Lorenz [1]. However, experimental test results indicate that cylinders buckled at loads well bellow those predicted by the early theoretical solutions based on small deflection theory. Subsequently, Donnell [2] realised that linear stability analyses were inadequate and suggested the need for a method of analysis that would account for large deflections. The first accurate solution for geometrically non-linear analysis was obtained by Von Karman and Tsien [3]. A few years later, the investigation carried out by Donnell and Wan [4] was of paramount importance, showing that initial imperfections are responsible for the great inconsistency between analytical estimates and experimental results. Since then, numerous studies of axially compressed CHS members have been carried out, in particular, for short and very short members (cylinders). Similarly, the buckling behaviour of CHS members under bending has long been studied. Analytical solutions to obtain the critical buckling moment of CHS beams were first developed and presented by Flügge [5,6], Seide and Weingarten [7] and Murray and Bilston [8]. One should also mention the numerical investigations carried out by Stephens et al. [9] and Karyadi [10]. More recently, an interesting state-of-the-art review of most of these studies was presented by Teng [11] and closed-form solutions have been developed by Elchalakani et al. [12]. Finally, the buckling behaviour of CHS members under torsion has also been studied for a long period of time. Since the pioneering work of Schwerin [13], several investigations were carried out and deserve to be mentioned, namely the works by Donnell [14], Batdorf et al. [15], Gerard [16], Schilling [17] and Yamaki and Matsuda [18]. Although in the context of heterogeneous and orthotropic materials, the buckling behaviour of

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cylindrical shells and tubes under torsion still attracts the interest of nowadays investigations, as it is confirmed by the recent works of Meyer-Piening et al. [19], Bisagni and Cordisco [20] and Tafreshi and Bailey [21].

Generalised beam theory (GBT), which extends Vlasov's classical beam theory in order to account for in-plane (local) flexural and distortional cross-section deformations, has been shown to constitute a rather powerful, elegant and clarifying tool to investigate the local and global buckling behaviour of thin-walled prismatic structural members. GBT was originally developed by Schardt [22], about two decades ago and in the context of isotropic materials and "unbranched" thin-walled cross-sections (see Fig. 1(a)). In the last decade, Davies [23] and, more recently, Camotim et al. [24] have applied extensively GBT to investigate the buckling behaviour cold-formed steel and composite thin-walled members. Besides accounting for the cross-section (local) deformations, which requires incorporating genuine folded-plate theory concepts, the most relevant feature exhibited by GBT resides in the fact that the member deformed configuration (buckling mode) is expressed as a linear combination of a set of pre-determined cross-section deformation modes. All these works were carried out in the context of members with unbranched cross-sections (see Fig. 1(a)). Very recently, Dinis et al. [25] developed, validated and illustrated an original GBT methodology to study the behaviour of members displaying arbitrary branched cross-sections (see Fig. 1(b)).

As far as the author knows, the only attempt to extend GBT to analyse the structural behaviour of thin-walled members with circular hollow cross-sections (see Fig. 1(c)) is due to Schardt [26] and Schardt [22]. However, these early attempts were developed mainly for first-order (linear) analysis and were not complemented by further investigations (for instance, these works do not include axisymmetric and/or torsional buckling). Therefore, the main objectives of this paper are the following: (i) to develop, validate and illustrate a methodology to perform a GBT analysis of circular hollow cross-sections, (ii) to present and discuss some numerical results concerning the local and global buckling behaviour of thin-walled columns (compression members), beams (flexural members), beam-columns and shafts (torsion members) and (iii) to revisit some classical buckling formulae for cylindrical shells obtained several decades ago. In particular, in-depth studies concerning the influence of member length on the variation of the critical stress and corresponding buckling mode shape are presented. Finally, and in order to validate and illustrate the application of the developed GBT, the results obtained are compared with values yielded by shell finite element analyses or values available in literature.

2. Formulation

Let us consider the CHS depicted in Fig. 2, with radius r and wall thickness t, and the global coordinate system X, Y, Z. In order to account for cross-section in-plane deformation effects, it is better to consider the local coordinate system x, θ and z, (longitudinal coordinate $x \in [0;L]$, angular coordinate $\theta \in [0;2\pi]$, thickness coordinate $z \in [-t/2;\ +t/2]$). Therefore, the displacement components in the local coordinate system (x, θ and z) are u, v and w, which correspond to *warping*, *transverse* and *flexural* displacements, respectively.

According to the theory of shells, the *complete* and *exact* strain-displacement (kinematic) relations [27,28] that satisfy the Love–Kirchhoff assumption are given by

$$\varepsilon_{xx} = u_{,x} + \tfrac{1}{2}\beta_x^2 + \tfrac{1}{2}\beta^2 + z\kappa_{xx}, \tag{1}$$

$$\varepsilon_{\theta\theta} = \frac{v_{,\theta} + w}{r} + \tfrac{1}{2}\beta_\theta^2 + \tfrac{1}{2}\beta^2 + z\kappa_{\theta\theta}, \tag{2}$$

$$\gamma_{x\theta} = \frac{u_{,\theta}}{r} + v_{,x} + \beta_x\beta_\theta + z\kappa_{x\theta}, \tag{3}$$

where

$$\beta_x = -w_{,x}, \tag{4}$$

$$\beta_\theta = \frac{v - w_{,\theta}}{r}, \tag{5}$$

$$\beta = \frac{1}{2}\left(v_{,x} - \frac{u_{,\theta}}{r}\right) \tag{6}$$

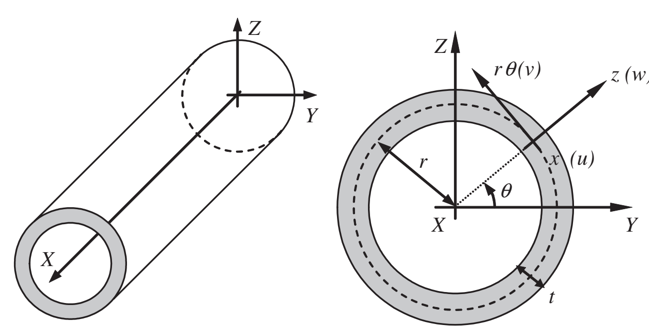

Fig. 2. Global and local coordinate system and displacement components.

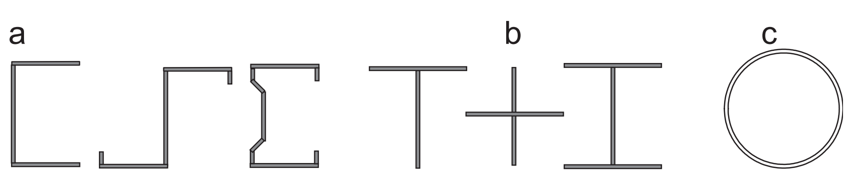

Fig. 1. (a) "Unbranched", (b) "branched", and (c) CHS configurations.