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Finite element formulation for inflatable beams

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Abstract

The discretized nonlinear equations for bending and buckling of inflatable beams are written by use of the virtual work principle with Timoshenko's kinematics, finite rotations and small strains. The linearized equations around a pre-stressed reference configuration are then deduced, giving rise to a new inflatable beam finite element. The stiffness matrix contains the shear coefficient and the internal pressure. Use is made of the particular 3-node beam element to investigate the bending and the buckling of a cantilever beam, the deflection of a pinched torus and the buckling of a torus submitted to a radial compressive force. The numerical results obtained with the beam element are shown to be close to analytical and three-dimensional (3D) membrane finite element results. The validity of the numerical results is discussed, in connection with the concepts of the crushing force or the wrinkling pressure of the inflated beam.

Keywords: Inflatable beam; Thin-walled structure; Membrane structure; Follower force; Finite element

1. Introduction

Inflatable beam structures have several interesting properties: they are lightweight and easily folded. The internal pressure yields tensile pre-stressing in the fabrics and implies a significant bearing capacity, especially when the pressure is high. First analytical studies on inflatable beams are not new. Comer and Levy [1] studied an inflatable cantilever beam using the usual Euler–Bernoulli kinematics. Fichter [2] gave the first solution in which the internal pressure appears in the deflection expressions. Main et al. [3,4] made experiments on a cantilever and improved Comer's theory. More recently, Wielgosz and Thomas [5,6] have derived analytical solutions for inflated panels and tubes by using the Timoshenko kinematics and by writing the equilibrium equations in the deformed state of the beam in order to take into account the follower force effect of the internal pressure. They have obtained new expressions for the deflection and shown that the limit load is proportional to the applied pressure and that the deflections are inversely proportional to the material properties of the fabrics and to the applied pressure, thus improving Fichter's theory. Lately, Le van and Wielgosz [7] have used the virtual work principle with finite displacements and rotations in order to derive the nonlinear equations for inflatable beams; the linearized equations once again improve Fichter's theory.

Since analytical solutions are only viable for isostatic beams, finite elements are required for computing beam networks. Their development seems to be a recent topic of research; Wielgosz and Thomas [8] have constructed a finite element devoted to inflatable panels. This element including the inflation pressure effect was obtained by the equilibrium finite element method, according to which the global compliance is the sum of the yarn and beam compliances. This element has been converted in a displacement one in order to be easily used in usual finite element softwares. A finite element for tubes can also be found in [6], where the equilibrium method has once again been used, giving rise to a non-symmetrical stiffness matrix depending on the pressure, which is suitable for lightly ventilated structures.

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The aim of this paper is to derive a new displacement finite element for inflatable beams directly from the virtual work principle. The result will be a symmetrical stiffness matrix which holds for airtight tubes.

In the first section of the paper, the finite element is derived for the static in-plane stretching and bending problem of an inflated cylindrical beam made of a membrane. First, the discretized nonlinear equations will be written by use of the virtual work principle with Timoshenko's kinematics, finite rotations and small strains. The procedure will enable one to correctly account for the shear effect and the pressure in the governing equations. Then, the tangent stiffness matrix will be derived as the sum of a tangent matrix due to the internal strain work and another tangent matrix due to the internal pressure. Whereas it is essential to assume finite rotations in order to correctly exhibit all the terms due the internal pressure, it is sensible to use the inflatable beam in small deformations only. Thus, the nonlinear problem will be linearized around a well-defined reference configuration. This will lead to the inflatable beam finite element involving the Timoshenko shear coefficient and the inflation pressure. The particular case of a 3-node beam finite element with quadratic shape functions will be considered.

The 3-node beam finite element will be used in the next sections to deal with the bending and the buckling of a cantilever inflatable beam, the bending of a pinched toroidal beam and the buckling of a toroidal beam submitted to a radial compressive force. In every case, the finite element results will be compared with the analytical or 3D membrane finite element results, showing the good performance of the proposed finite element. Also, the validity of the numerical results will be discussed, in connection with the concepts of the crushing force or the wrinkling pressure of the inflated beam.

The formulation and results presented in this paper are the finite element counterpart of Ref. [7].

2. The finite element for an inflatable beam

Consider an inflatable beam made of a cylindrical membrane undergoing axial stretch and bending in the xy-plane (Fig. 1) under the combined action of an internal pressure and other external dead loads. In the reference (or initial) configuration, the length of the beam is L_o, the cross-section area S_0, the second moment of area I_0, and all the centroids G_o of the cross-sections lie on the x-axis. The reference thickness of the membrane is h_o.

Remark. For inflatable structures, it should be emphasized that the loading is applied in two successive stages: first, the beam is inflated to pressure p, and then the other external forces are applied. At the beginning of the first stage, the internal pressure is zero and the beam is in a *natural* (or *stress free*) state. On the other hand, the reference configuration, which corresponds to the end of the first stage (before external forces other than the internal pressure are applied), is in a *pre-stressed* state. To clearly distinguish between the two states, we shall use index $\varnothing$ to denote the quantities in the *natural* state, as opposed to the usual index o for quantities related to the *reference* configuration. Thus, $L_\varnothing$ and $h_\varnothing$ designate the natural length and thickness; while L_o and h_o the reference ones. Computing L_o and h_o corresponding to a given pressure may be difficult depending on the problem at hand, but this is an independent subject which actually is common to every problem with a pre-stressed reference configuration.

In order to derive the discretized equations for the inflatable beam, use will be made of the principle of virtual work in the three-dimensional (3D) Lagrangian form

$$\forall \text{ virtual displacement field } \mathbf{V}^*,$$
$$-\int_{\Omega_0} \mathbf{\Pi}^{\mathrm{T}} : \operatorname{grad} \mathbf{V}^* \, \mathrm{d}\Omega_0 + \int_{\Omega_0} \mathbf{f}_o \cdot \mathbf{V}^* \, \mathrm{d}\Omega_0 + \int_{\partial\Omega_0} \mathbf{V}^* \cdot \mathbf{\Pi} \cdot \mathbf{N} \, \mathrm{d}S_0 = 0, \tag{1}$$

where Ω_0 is the 3D region occupied by the beam in the reference configuration, $\partial\Omega_0$ its boundary, $\mathbf{\Pi} = \mathbf{F\Sigma}$ and $\mathbf{\Sigma}$ the first (non-symmetric) and second (symmetric) Piola–Kirchhoff stress tensors ($\mathbf{F}$ is the deformation gradient), $\mathbf{f}_o$ the body force per unit reference volume and $\mathbf{N}$ the unit outward normal in the reference contribution. Following the Timoshenko beam model, the displacement of any material point $\mathbf{P_o}(X, Y, Z)$ in the beam is given by:

$$\mathbf{U}(\mathbf{P_o}) = \mathbf{U} + \mathbf{R}(\theta) \cdot \mathbf{G_o P_o}, \tag{2}$$

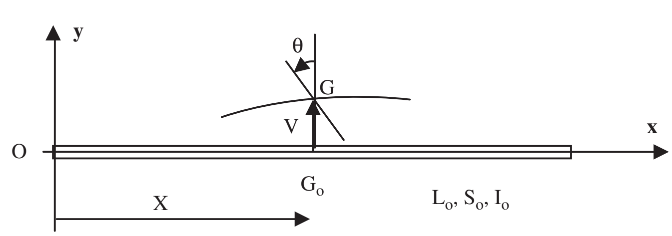

Fig. 1. The inflated beam model.