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Dynamic stability analysis of composite skew plates subjected to periodic in-plane load

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Abstract

Here, the dynamic stability characteristics of simply supported laminated composite skew plates subjected to a periodic in-plane load are investigated using the finite element approach. The formulation includes the effects of transverse shear deformation, in-plane and rotary inertia. The boundaries of the instability regions are obtained using the Bolotin's method and are represented in the non-dimensional load amplitude-excitation frequency plane. The principal and second instability regions are identified for different parameters such as skew angle, thickness-to-span ratio, fiber orientation and static in-plane load.

Keywords: Composite skew plate; Dynamic stability; Static in-plane load

1. Introduction

The increased utilization of composite materials in thin-walled structural components of aircrafts, submarines, automobiles and other high-performance application areas have necessitated a strong need to understand their dynamic characteristics under different loading conditions. These structural components, i.e., beams, plates and shells are sometimes subjected to periodic in-plane load and become dynamically unstable (with transverse vibration increases without bound) for certain combinations of load amplitude and disturbing frequency. The above phenomenon is called dynamic instability or parametric resonance. A general theory of dynamic stability of elastic structures is available in the text book of Bolotin [1].

A number of researchers [2–8] have investigated the dynamic stability characteristics of rectangular composite plates using Bolotin's method, where, periodic solutions in the form of Fourier series are employed, and the boundaries of the instability regions are obtained using eigenvalue approach. It is observed from the existing literature that the primary instability region that occurs in the vicinity of $2\omega_1$ (ω_1—the lowest natural frequency) of rectangular plates has received considerable attention of the researchers [2–6], whereas, limited work [7–8] has been focused to study the other dynamic instability regions. Furthermore, most of the earlier research works assumed only one term (first-order) Fourier series approximation, which may not represent the boundaries correctly. Moorthy et al. [6], Srinivasan and Chellapandi [7], Chattopadhyay and Radu [8] have studied the effect of first- and second-order approximations on the dynamic instability boundaries of rectangular composite plates. Significant deviations are observed between the first- and second-order approximations in the result of Srinivasan and Chellapandi [6] and Chattopadhyay and Radu [7] for the case of clamped composite plates. However, similar work on dynamic stability analysis of non-rectangular plates appears to be scarce in the literature. Baldinger et al. [9] considered polygonal plates to investigate the first two instability regions for various loading conditions.

The plates with non-rectangular plan-forms like skew plates find wide application in the aerospace industry. Vibration and stability analyses of such structures, have recently gained importance among researchers [10–14]. Merrit and Willems [15] investigated the boundaries of dynamic instability regions of stiffened skew isotropic

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plates. Young and Chen [16] studied the effect of in-plane forces on the critical aerodynamic pressure of isotropic skew plates. Liao and Cheng [17] studied the dynamic instability characteristics of stiffened isotropic and composite square plate, skew plate and cylindrical shells using 3-dimensional degenerated shell and beam element. However, to the best of author's knowledge, dynamic stability analysis of composite skew plates is not yet commonly available in the literature.

In the present paper, a four-noded shear flexible quadrilateral high-precision plate-bending element developed recently [13,14] is extended to analyze the dynamic stability behavior of perfect laminated composite skew plates. As the element is free from locking phenomenon, all the energy terms are evaluated using full numerical integration scheme. The formulation includes in-plane and rotary inertia effects. Periodic solutions in the form of Fourier series are employed, and the boundaries of the principal and second instability regions are obtained using eigenvalue approach. Higher-order approximation of the Fourier series is assumed for converged result. A detailed parametric study is carried out to study the influences of the skew angle, lay-up, and static in-plane load on the dynamic stability characteristics of simply supported laminated skew plates. >

2. Formulation

Fig. 1(a) shows the rectangular Cartesian co-ordinate system along with the associated covariant base vectors ($\mathbf{g_1}$, $\mathbf{g_2}$, $\mathbf{g_3}$) and contravariant base vectors ($\mathbf{^1g}$, $\mathbf{^2g}$, $\mathbf{^3g}$) for the skew plate having a and b as the length and width, and ψ as the skew angle. The covariant components of the displacement vector $\mathbf{u}$ ($\mathbf{u} = u_1\mathbf{^1g} + u_2\mathbf{^2g} + u_3\mathbf{^3g}$) of a shear deformable plate can be expressed in terms of the contravariant components of the position vector $\mathbf{r}$ ($\mathbf{r} = {}^1r\mathbf{g_1} + {}^2r\mathbf{g_2} + {}^3r\mathbf{g_3}$) [13,14] as

$$\begin{aligned} u_1\left({}^1r, {}^2r, {}^3r\right) &= u_1^0\left({}^1r, {}^2r\right) + {}^3r\,{}^1\phi\left({}^1r, {}^2r\right), \\ u_2\left({}^1r, {}^2r, {}^3r\right) &= u_2^0\left({}^1r, {}^2r\right) + {}^3r\,{}^2\phi\left({}^1r, {}^2r\right), \\ u_3\left({}^1r, {}^2r, {}^3r\right) &= u_3\left({}^1r, {}^2r\right), \end{aligned} \tag{1}$$

where

$$\begin{aligned} {}^1\phi\left({}^1r, {}^2r\right) &= \left\{\gamma_1\left({}^1r, {}^2r\right) - u_{3,1}\right\}, \\ {}^2\phi\left({}^1r, {}^2r\right) &= \left\{\gamma_2\left({}^1r, {}^2r\right) - u_{3,2}\right\}. \end{aligned}$$

Here, ${}^1\phi$ (${}^1r$, ${}^2r$) and ${}^2\phi$ (${}^1r$, ${}^2r$) are the total rotations; $\gamma_1({}^1r, {}^2r)$ and $\gamma_2({}^1r, {}^2r)$ are the rotations due to shear deformation of the normal to the plate middle surface around $\mathbf{^2g}$ axis and $\mathbf{^1g}$ axis, respectively; and $(\ )_{,i}$ represents the partial differentiation of the variable preceding it with respect to ${}^ir$.

Following standard procedure, the finite element equations for the plate (Fig. 1b) under uniaxial compressive force $F(t) = N_0 + N_1 \cos\theta t$ are derived as

$$[\mathbf{M}]\left\{\ddot{\delta}\right\} + [\mathbf{K}_L + (N_0 + N_1 \cos\theta t)\mathbf{K}_G]\{\boldsymbol{\delta}\} = \{0\}$$

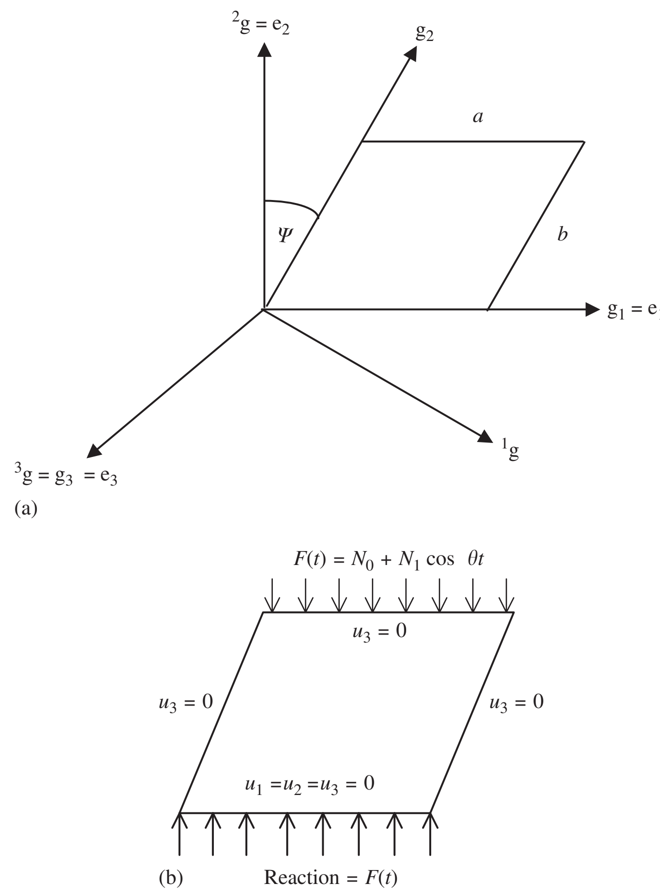

Fig. 1. (a) Oblique co-ordinate system for the skew plate, and (b) simply supported skew plate under uni-axial periodic force.

or

$$[\mathbf{M}]\left\{\ddot{\delta}\right\} + [\mathbf{K}^* + N_1 \cos\theta t\mathbf{K}_G]\{\boldsymbol{\delta}\} = \{0\}, \tag{2}$$

where $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{K}_G$ are mass, linear stiffness and geometric stiffness matrices, respectively and δ is the vector of degrees of freedom and $\mathbf{K}^* = \mathbf{K}_L + N_0\mathbf{K}_G$. Eq. (2) represents the dynamic stability problem of a plate subjected to a periodic in-plane force. The governing Eq. (2) is solved using finite element approach based on C^1 continuous element developed recently [13,14].

A four-noded quadrilateral plate bending element with 10 degrees of freedom per node, namely $u_1^0, u_2^0\ u_3, u_{3,1}, u_{3,2}, u_{3,11}, u_{3,12}, u_{3,22}, \gamma_1$ and γ_2 are used here. The linear polynomial shape functions are employed to describe the field variables corresponding to in-plane displacements (u_1^0, u_2^0) and rotations due to shear of the middle surfaces (γ_1, γ_2), whereas quintic polynomial function is considered for the lateral displacement (u_3) and are expressed as follows:

$$u_1^0 = c_k\left({}^1r\right)^i\left({}^2r\right)^j;\ \ i,j = 0,1 \quad \text{and} \quad k = 1,4,$$

$$u_1^0 = c_k\left({}^1r\right)^i\left({}^2r\right)^j;\ \ i,j = 0,1 \quad \text{and} \quad k = 5,8,$$

$$u_1^0 = c_k\left({}^1r\right)^i\left({}^2r\right)^j + c_k\left({}^1r\right)^m\left({}^2r\right)^n + c_k\left({}^1r\right)^n\left({}^2r\right)^m;$$
$$i,j = 0,1,2,3;\ \ m = 0,1;\ \ n = 4,5;\ \text{and}\ \ k = 9,32,$$