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Crack analysis in orthotropic media using the extended finite element method

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Abstract

An extended finite element method has been proposed for modeling crack in orthotropic media. To achieve this aim a discontinuous function and two-dimensional asymptotic crack-tip displacement fields are used in a classical finite element approximation enriched with the framework of partition of unity. It allows modeling crack by standard finite element method without explicitly defining and re-meshing of surfaces of the crack. In this study, fracture properties of the models are defined by the mixed-mode stress intensity factors (SIFs), which are obtained by means of the domain form of the interaction integral (*M*-integral). Numerical simulations are performed to verify the approach, and the accuracy of the results is discussed by comparison with other numerical or (semi-) analytical methods.

Keywords: Extended finite element method (X-FEM); Orthotropic material; Stress intensity factor (SIF); Interaction integral; *M*-integral; Crack

1. Introduction

According to the wide application of orthotropic materials, such as composites, in various structural systems like those in aerospace and automobile industries, power plants, etc., analyzing and modeling such materials have been among the most interesting topics of research in recent decades. The main advantages of using these materials can be attributed to their high stiffness and low ratio of weight to strength in comparison to other materials.

In the analytical field, Sih et al. [1], Bogy [2] Bowie and Freese [3], Barnett and Asaro [4] and Kuo and Bogy [5] have worked on finding the stress and displacement fields around a linear crack in an anisotropic medium. More advanced contributions can be found in Carloni et al. [6,7] and Nobile and Carloni [8].

There are many numerical methods available for analyzing orthotropic composites such as finite element method (FEM), finite difference method (FDM), and meshless methods. FEM has been one of the most powerful tools for numerical simulations in the past half a century. Finite element approximation can be significantly improved in modeling discontinuity when enriched using the framework of partition of unity [10,11]. In this method a discontinuous function and the near-tip asymptotic displacement fields are added to the finite element approximation by means of partition of unity. Belytschko and Black [12], Moës et al. [13], Dolbow et al. [14–17], Dolbow and Nadeau [18], Daux et al. [19], reported 2-D isotropic modelings, while Sukumar et al. [20] conducted 3-D modeling.

In this paper, the method is extended to orthotropic media. Near-tip asymptotic displacement field is based on the work by Carloni et al. [8]. Stress-intensity factors are evaluated via a form domain of interaction integral proposed by Kim and Paulino [21] for homogeneous orthotropic materials.

2. Mechanics of orthotropic materials

In an extended FEM, near-tip displacement fields are required for modeling the crack. Here, the analytical

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displacement fields derived by Carloni et al. [6,7] and Nobile and Carloni [8] are used.

Consider there is a crack in an orthotropic medium and the orthotropic body is subjected at infinity to a uniform biaxial load (T and kT), applied along X- and Y-directions. (Fig. 1)

The displacement field can be written as [7]

$$
\begin{aligned}
u = {} & \frac{2\beta}{C_{66}(p_1-p_2)}\sqrt{2lr} \\
& \times\left\{T_2\left[\frac{p_2\sqrt{g_2(\theta)}}{l_2(\alpha-p_2^2)}\cos\frac{\theta_2}{2}-\frac{p_1\sqrt{g_1(\theta)}}{l_1(\alpha-p_1^2)}\cos\frac{\theta_1}{2}\right]\right. \\
& \left.+p_1p_2T_3\left[\frac{\sqrt{g_2(\theta)}}{l_2(\alpha-p_2^2)}\sin\frac{\theta_2}{2}-\frac{\sqrt{g_1(\theta)}}{l_1(\alpha-p_1^2)}\sin\frac{\theta_1}{2}\right]\right\} \\
& -\frac{2\beta p_1p_2(T_2-p_1p_2T_1)}{C_{66}l_1l_2(\alpha-p_1^2)(\alpha-p_2^2)}(l+r\cos\theta) \\
& -\frac{\beta T_3(p_1+p_2)^2}{C_{66}l_1l_2(\alpha-p_1^2)(\alpha-p_2^2)}\,r\sin\theta,
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
v = {} & \frac{1}{C_{66}(p_1-p_2)}\frac{\sqrt{2lr}}{l_1l_2} \\
& \times\left\{T_2\left[l_1\sqrt{g_2(\theta)}\sin\frac{\theta_2}{2}-l_2\sqrt{g_1(\theta)}\sin\frac{\theta_1}{2}\right]\right. \\
& \left.+T_3\left[l_2p_2\sqrt{g_1(\theta)}\cos\frac{\theta_1}{2}-l_1p_1\sqrt{g_2(\theta)}\cos\frac{\theta_2}{2}\right]\right\} \\
& +\frac{T_3(p_1+p_2)(l_1-l_2)}{2C_{66}l_1l_2(p_1-p_2)}(l+r\cos\theta) \\
& +\frac{(T_2-p_1p_2T_1)}{C_{66}(p_1^2-p_2^2)}\left(\frac{p_2}{l_1p_1}-\frac{p_1}{l_2p_2}\right) \\
& \times\frac{\beta T_3(p_1+p_2)^2}{C_{66}l_1l_2(\alpha-p_1^2)(\alpha-p_2^2)}\,r\sin\theta,
\end{aligned}
\tag{2}
$$

where l_1, l_2, α and β are coefficients material properties [7] and C_{ij} $(i,j=1,2,6)$ are constitutive coefficients,

$$
\begin{aligned}
p_1 &= \left(A-\left(A^2-\frac{C_{22}}{C_{11}}\right)^{1/2}\right)^{1/2}, \\
p_2 &= \left(A+\left(A^2-\frac{C_{22}}{C_{11}}\right)^{1/2}\right)^{1/2},
\end{aligned}
\tag{3}
$$

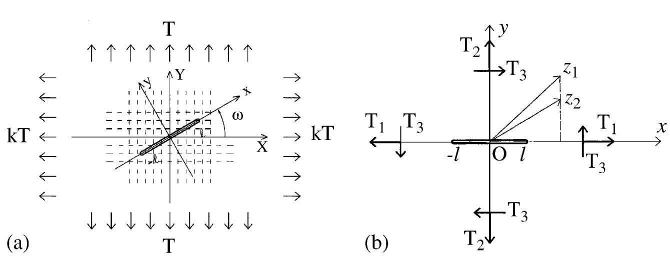

Fig. 1. Crack geometry and implied tensile stresses: (a) an inclined crack subjected to biaxial loading [8]; (b) local stress components at infinity for the inclined crack [7].

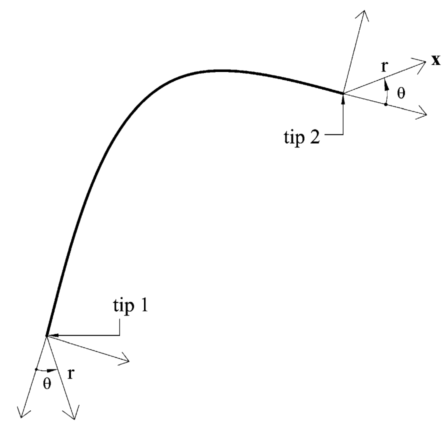

Fig. 2. Local co-ordinates at both crack-tips.

$$
A=\frac{1}{2}\left[\frac{C_{66}}{C_{11}}+\frac{C_{22}}{C_{66}}-\frac{(C_{12}+C_{66})^2}{C_{11}C_{66}}\right],
\tag{4}
$$

$$
g_j(\theta)=\left(\cos^2\theta+\frac{\sin^2\theta}{p_j^2}\right)^{1/2},\quad j=1,2,
\tag{5}
$$

$$
\theta_j=tg^{-1}\left(\frac{y}{p_jx}\right)=tg^{-1}\left(\frac{tg\theta}{p_j}\right),
\tag{6}
$$

where r and θ are the polar co-ordinates and x and y are the Cartesian co-ordinates in the local co-ordinate system at each crack-tip (see Fig. 2).

It should be noted that Eqs. (1) and (2) are only related to the cases where A in Eq. (4) and A^2-C_{22}/C_{11} in Eq. (3) are positive, therefore, p_1 and p_2 become real positive numbers, as discussed by Carloni and Nobile [6], Carloni et al. [7] and Nobile and Carloni [8]; applicable for most composites [9].

3. Extended FEM

X-FEM was originally proposed by Belytschko and Black [12] and Dolbow [14] and later modified and applied to various crack analysis problems by Daux et al. [19] and Sukumar et al. [20]. A numerical X-FEM model is constructed by dividing the model into two parts; first generating a mesh for the domain geometry (neglecting the existence of any crack or other discontinuities) and the second part is enriching finite element approximation by appropriate functions for modeling any imperfections. In the following sections the basics of X-FEM formulation is utilized.

3.1. Basic formulation

Consider $\mathbf{x}$ is a point of $\mathbf{R^2}$ (for 2-D space) or $\mathbf{R^3}$ (for 3-D space) in the finite element model and $\mathbf{N}$ is a set of nodes defined as $\mathbf{N}=\{n_1,n_2,\ldots,n_m\}$, m is the number of nodes in an element. The enriched approximation of displacement