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Lower bound buckling strength of axially loaded sandwich cylindrical shell under lateral pressure

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Abstract

Effects of the lateral pressure on the FEM and reduced stiffness lower bound buckling strength of axially loaded sandwich cylindrical shell are examined. Further, a reduced stiffness lower bound buckling strength for the axially loaded sandwich cylindrical shell under lateral pressure is proposed. The effect of the lateral pressure on the FEM and reduced stiffness lower bounds are corresponding; it causes them to reduce slightly. However, reduced stiffness buckling mode shape remains the same. In addition, the proposed reduced stiffness lower bound buckling strength is shown to provide effective and valid for cores having different shear stiffness. It provides comparatively close lower bounds to short axially loaded sandwich cylindrical shells under lateral pressure. Further, it provides a safe lower bound that does not depend on precise geometrical imperfection spectra and lateral pressure and it is simple and easy to employ.

Keywords: Sandwich shell; Lower bound; Buckling; Axial loading; Lateral pressure

1. Introduction

The axially loaded cylindrical shell is well known for its high sensitivity to the initial geometric imperfections, which can dramatically reduce its buckling strength [1–5]. Under these circumstances, a reliable and accountable method for the design of cylindrical shell arises. As a result, the reduced stiffness lower bound buckling strength for the imperfection sensitive cylindrical shell has been proposed and its validity has already been shown in publications by Croll and Yamada et al. [1–3,7,8].

Further, the same findings of geometrical imperfection sensitivity also hold true for axially loaded sandwich cylindrical shell. This has been demonstrated in publications by Ohga et al. [5,6,9]. Accordingly, the reduced stiffness lower bound buckling strength of the geometrical imperfection sandwich cylindrical shells has been proposed and its validity has already been verified [5,6,9]. The proposed reduced stiffness lower bound buckling strength not only provides safe lower bounds that do not depend on precise imperfection spectra but also it is simple and easy to employ.

However, effects of lateral pressure on the reduced stiffness buckling strength of the axially loaded sandwich cylindrical shell are still not found publicized. Therefore, an attempt is made in this paper to examine such effects. Further, a reduced stiffness lower bound buckling strength, which can avoid effects and consequence of not only geometrical imperfects but also lateral pressures, is proposed for the axially loaded sandwich cylindrical shell. It is compared with that 'FEM lower bound buckling strength' [5,6,9] to verify its validity and effectiveness. The 'FEM lower bound buckling strength' is obtained by FEM analysis of selected numerical examples. The proposed lower bound buckling strength not only avoids effects and consequences of geometric imperfections but also that of lateral pressure. Further, it does not depend on precious imperfection spectra and it is simple and easy to employ.

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Notations

a	mean radius of the sandwich cylinder
A_i	amplitude of the displacement function
C_{ij}	coefficients of the eigenmatrix
E_f	Young's module of the face material
E_x^F, E_s^F, E_{xs}^F	fundamental state strains
G_c	shear modulus of core material
G_f	shear modulus of face material
h	total thickness of the sandwich cylinder
h_c	thickness of the core
h_f	thickness of the face
L	length of the sandwich cylindrical shell
m	number of axial half-waves in critical mode
M_x^F, M_s^F, M_{xs}^F	fundamental state moments
n	number of circumferential half-waves in critical mode
N_x^F, N_s^F, N_{xs}^F	fundamental state stresses
p	axial load
q_0	lateral pressure
u, v, w	displacements
v_f	Poisson's ratio of face material
Π	total potential energy
Π_0, Π_1, Π_2	constant, linear, quadratic components of the total potential energy
σ	axial stress
σ_x, σ_s	incremental axial and circumferential stress
τ_{xs}, $\tau_{x\zeta}$, $\tau_{s\zeta}$	incremental shear stresses
α	circumferential wavelength parameter (n/a)
ρ	non-dimensional axial wavelength parameter $(m\pi/l)$
ε_x, ε_s	incremental axial and circumferential strains
β_x, β_s	rotations about s and x axes
γ_{xs}, $\gamma_{x\zeta}$, $\gamma_{s\zeta}$	incremental shear strains

Subscripts and superscripts

c	belonging to classical model
fem	belonging to FEM analysis
rs	belonging to reduced stiffness model
$'$	stresses and strains linearly dependant on displacements
$''$	stresses and strains quadratically dependant on displacements

2. Reduced stiffness lower bound buckling strength

2.1. Essential equations and boundary conditions

A simply supported sandwich cylindrical shell of length L, mean radius a, face thickness h_f and core thickness h_c (Fig. 1) is considered. The core of the shell is assumed to support only transverse shear as shown in Fig. 2. In contrast, on the assumption of thin shell, the face sheets are assumed not to support transverse shear (Fig. 2).

By employing the sine convention and the notations given in Figs. 1 and 2, the linear incremental membrane stress resultants σ_x', σ_s', τ_{xs}', $\tau_{x\zeta}'$ and $\tau_{s\zeta}'$ in the given directions and planes can be written by

$$\sigma_x' = D_f(\varepsilon_x' + v_f\varepsilon_s'), \tag{1a}$$

$$\sigma_s' = D_f(\varepsilon_s' + v_f\varepsilon_x'), \tag{1b}$$

$$\tau_{xs}' = G_f\gamma_{xs}', \tag{1c}$$

$$\tau_{x\zeta}' = G_c\gamma_{x\zeta}', \tag{1d}$$

$$\tau_{s\zeta}' = G_c\gamma_{s\zeta}'. \tag{1e}$$

Here, the shear modulus of the core is denoted by G_c while that of face is taken as G_f. In addition, D_f equals $E_f/(1-v_f^2)$. E_f and v_f denote the Young's module and Poisson's ratio of the face, respectively. The relevant linear incremental strains defined by the orthogonal curvilinear coordinate system (x, s, ζ) shown in Fig. 1 can be written by

$$\varepsilon_x' = \frac{\partial u}{\partial x} + \zeta\frac{\partial \beta_x}{\partial x}, \tag{2a}$$

$$\varepsilon_s' = \frac{\partial v}{\partial s} + \frac{w}{a} + \zeta\frac{\partial \beta_s}{\partial s}, \tag{2b}$$

$$\gamma_{xs}' = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial s} + \zeta\left(\frac{\partial \beta_s}{\partial x} + \frac{\partial \beta_x}{\partial s}\right), \tag{2c}$$

$$\gamma_{x\zeta}' = \frac{\partial w}{\partial x} + \beta_x, \tag{2d}$$

$$\gamma_{s\zeta}' = \frac{\partial w}{\partial s} - \frac{v}{a} + \beta_s. \tag{2e}$$

Here, u, v and w denote the displacements in the x, s and ζ directions, respectively (Fig. 1). β_x and β_s symbolize the rotations with respect to the s and x axes as can be seen in Fig. 1. Further, the non-linear stresses can be written by

$$\sigma_x'' = D_f(\varepsilon_x'' + v_f\varepsilon_s''), \tag{3a}$$

$$\sigma_s'' = D_f(\varepsilon_s'' + v_f\varepsilon_x''). \tag{3b}$$

These stresses associate with the non-linear strain components:

$$\varepsilon_x'' = \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = (w_x')^2, \tag{4a}$$