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Theoretical prediction and numerical simulation of multi-cell square thin-walled structures

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Abstract

The axial crushing of square multi-cell columns were studied analytically and numerically. Based on the Super Folding Element theory, a theoretical solution for the mean crushing force of multi-cell sections were derived by dividing the profile into 3 parts: corner, crisscross, and T-shape. Numerical simulations of square multi-cell sections subjected to dynamic axial crushing were conducted and an enhancement coefficient was introduced to account for the inertia effects for aluminum alloy AA6060 T4. The analytical solutions show an excellent agreement with the numerical results. It was found that the crisscross part was the most efficient component for energy absorption and the energy absorption efficiency of a single-cell column can be increased by 50% when the section was divided into 3×3 cells. Finally, the proposed method was extended to analyze the plateau stress of square cell honeycomb subjected to out-plane axial crushing and to some extent validate the mechanical insensitivity of honeycomb to cell size.

Keywords: Multi-cell column; Energy absorption; Axial crushing; Honeycomb

1. Introduction

Thin-walled metal tubes, particularly those of square or circular cross-section, are widely used as energy absorbers since they are relatively cheap and efficient for absorbing energy. The behaviors of structural collapse in axial crushing of them have been extensively studied over the past decades. The axial crushing of circular tube was firstly analyzed by Alexander [1] and found to be an excellent mechanism for energy absorption. Also, an approximate theoretical expression is developed to derive the average crushing force. Then experimental research and theoretical predictions on axial crushing of circular and square tubes either statically or dynamically are detailed by Wierzbicki and Abramowicz [2,3], Abramowicz and Jones [4,5], Andrews et al.[6] and others. The general characteristics of the force–displacement curves of circular tube and square tube are similar: the axial force first reaches an initial peak, followed by a drop and then fluctuates. However, their collapse modes are very different. For a circular tube, it can collapse with the concertina mode, diamond mode, mixed mode or global buckling, which depends primarily on the ratio of dimensions of the tube, namely length, diameter and thickness. For a square tube, symmetric mode, asymmetric mode, extensional mode or global buckling may appear during axial compression.

While hollow tubes were still addressed by various authors (e.g. Guillow et al.[7], Langseth et al.[8,9], Huang and Lu [10], Tarigopula et al.[11]), recently tubes filled by cellular metal material are becoming the focus of many researchers and a number of authors have contributed to this topic. Because of low density and nearly constant stress until compressed to a high densified strain, cellular materials such as aluminum honeycomb or foam, has the potential for increasing energy absorption of thin-walled structures. The main mechanisms providing enhancements (compared to an empty tube) are the compression of the filler material itself and interaction effects between the filler and tube. Seitzberger et al. [12] have shown that the energy absorption characteristics of thin-walled columns can be considerably improved by aluminum foam filling. Hanssen

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et al. [13,14] also studied the influence of aluminum foam to square and circular aluminum extrusions; Simple expressions on the mean crushing force and maximum crushing force were presented. Efficiency improvement is also shown when numerical method was adopted by Santosa and Wierzbicki [15] to study axially compressed square aluminum tubes, which were filled with honeycomb or foam.

In theory aspect, after a classical plastic folding analysis of a cylindrical shell under axial crush loading done by Alexander in 1960 [1], plastic deformation of prismatic columns was analyzed by the Super Folding Element theory proposed by Wierzbicki and Abramowicz [2,3]. In the Super Folding Element theory, by adopting a rigid-plastic material and using the condition of kinematic continuity on the boundaries between rigid and deformable zones, a model consisting of trapezoidal, toroidal, conical and cylindrical surfaces with moving hinge lines was analyzed. Main plastic energy dissipation mechanisms of typical crumpled thin-walled metal structures were included in the model. Experimental validation of the theory was then performed by Abramowicz and Jones [4].

The number of angle elements on a tube's cross-section decides, to a large extent, on the efficiency of the energy absorption [2,3]. It is therefore desirable to design thin-walled extrusion with multiple cells for weight-efficient energy absorption. For multi-cell columns, due to the complicacy of the problem, few literatures are available. Chen and Wierzbicki [16] presented closed-form solutions to calculate the mean crushing strength of single-cell, double-cell and triple-cell hollow aluminum profiles and corresponding foam-filled extrusion. The analytical solution for calculating the mean crushing force of multi-cell profiles with four square elements at the corner is derived by Kim [17]. Both solutions were shown to compare very well with the numerical results. Triggers were introduced to the numerical models of multi-cell extrusions in order to obtain stable and regular crushing modes.

The axial crushing of multi-cell aluminum extrusion as shown in Fig. 1 was studied analytically and numerically in this paper. Based on the proposed Super Folding Element theory [2,3], an analytical solution for mean crushing force of multi-cell square sections was developed, and it was very convenient to apply.

Nonlinear explicit finite element (FE) codes such as PAM-CRASH, ABAQUS Explicit and LS-DYNA are powerful tools in numerical simulations of the large deformation dynamic response of structures. A large amount of numerical work [11–17] has been carried out on thin-walled structures by these commercial codes apart from the above analytical and experimental investigations. LS-DYNA [18] was used to conduct the numerical simulations in this paper. The analytical solutions derived show an excellent agreement with the numerical results.

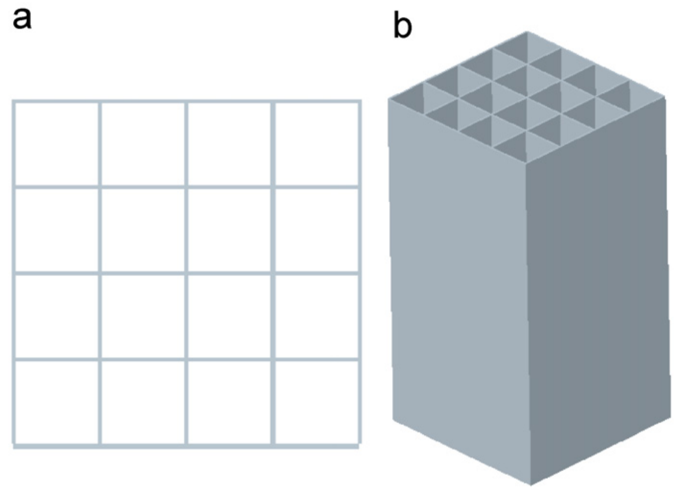

Fig. 1. Representation of a multi-cell column. (a) Top view, and (b) general view.

2. Theoretical analysis of multi-cell square columns

As mentioned above, Super Folding Element method was developed by Wierzbicki and Abramowicz [2] to predict the mean crushing force of thin-walled structures, and the theory was applied to the problem of progressive folding of a thin-walled rectangular column. For rigid–perfectly plastic material, the mean crushing force P_m can be calculated by

$$P_m = 9.56\sigma_0 b^{1/3} t^{5/3}, \tag{1}$$

where σ_0 denotes the flow stress of the material; b the sectional width and t the wall thickness. Abramowicz and Jones [5] later changed the constant 9.56 to 13.06.

To take strain hardening effects into account, the energy equivalent flow stress for material with power law hardening can be calculated by using [19]

$$\sigma_0 = \sqrt{\frac{\sigma_\mathrm{y}\sigma_\mathrm{u}}{1+n}}, \tag{2}$$

where σ_y and σ_u denote the yield strength and the ultimate strength, respectively; and n is the strain hardening exponent.

Chen and Wierzbicki [16] adopted a simplified approach to derive the analytical solution of mean crushing force of multi-cell sections. Rather than building the model consisting of trapezoidal, toroidal, conical and cylindrical surfaces with moving hinge lines, they proposed a basic folding element consisting of 3 extensional triangular elements and 3 stationary hinge lines. Expressions of mean crushing force of double cell and triple cell were given. They are $P_\mathrm{m} = 9.89\sigma_0 b^{1/2} t^{3/2}$, and $P_\mathrm{m} = 12.94\sigma_0 b^{1/2} t^{3/2}$, respectively. In their work, the membrane energy of the multi-cell structure was determined by the number of flange, the contribution of each flange was considered to be the same.

For a complete collapse of a single fold of a tube, considering the energy equilibrium of the system, the