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Ultimate strength of submarine pressure hulls with failure governed by inelastic buckling

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Abstract

The analysis of submarine pressure hull assumes great importance among structural engineers due to the complexity involved in the collapse mechanism of stiffened shell structures. In most of the cases, the failure of stiffened shell structures occurs due to elastic buckling. But for some combinations of shell-stiffener geometry and material characteristics, the structure can fail by inelastic buckling, for which the methods of analysis are meagre. In this paper, the analysis of submarine pressure hull structure in which the failure gets governed by inelastic buckling is demonstrated. Three different approaches have been employed to investigate the ultimate strength of the ring stiffened submarine pressure hull structure with inelastic buckling modes of failure. The methods used are 'Johnson–Ostenfeld inelastic correction', 'imperfection method' and 'finite element approach'. A typical submarine shell structure has been analysed for the inelastic buckling failure using these three approaches and the results are discussed.

Keywords: Inelastic buckling; Submarine pressure hulls; Ring stiffened shell; Johnson–Ostenfeld inelastic correction; Imperfection method; Finite element approach

1. Introduction

Submarine pressure hulls are commonly of the form of stiffened shell structures. Since, these shell structures have complicated buckling phenomena, their analysis is of greater importance in structural engineering field. Most of the stiffened shell structures fail in modes like ring buckling, interstiffener buckling, stiffener tripping and overall buckling. These elastic buckling modes and their analyses have been studied and discussed by various authors and from the researches it has been found that inelastic effects are an important factor in the buckling analysis of shells and have, therefore, motivated extensive studies of inelastic buckling.

Kendrick [1] has studied the elastic stability of externally pressurized vessels and derived the energy expressions and boundary conditions for ring stiffened cylinders. He has also discussed several buckling modes of ring stiffened shells. The design criteria of submarine structures in which various failure modes due to elastic buckling have been discussed by Kendrick [2]. He also attempted to describe the behaviour of such structures beyond the elastic limit. Bulson [3] has conducted several tests on underground cylindrical shells and studied the stability behaviour. He has surveyed the analytical methods for predicting the critical radial pressure to cause elastic buckling of soil-surrounded thin-walled cylinders and also tried to discuss some post-buckling analyses of shell structures. An idealization concept for stability analysis of ring reinforced cylindrical shells under external pressure has been discussed by Barlag and Rothert [4]. They have introduced a nomograph, which can be used to determine the ideal local and global buckling pressures and critical buckling form of a ring stiffened circular cylindrical shell under external pressure. Ross and Waterman [5] studied the inelastic instability behaviour of cylindrical shells by conducting experiments on six circular corrugated cylinders under external hydrostatic pressure. From the experimental results, they have developed a design chart suitable for designing the corrugated vessels to guard against inelastic instability. In spite of many years of efforts by different approaches to develop good prediction techniques for plastic and elastic–plastic buckling of stiffened shells, comprehensive and reliable methods are still scarce. Hence in the present study, a rigorous analysis of internally ring stiffened submarine pressure hull, in which the failure gets governed by inelastic buckling, has been demonstrated.

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Nomenclature

a	distance of plastic neutral axis from the centroid of the shell-stiffener geometry
d_w	depth of web
E	Young's modulus
f	flange width of stiffener.
I_{xx}	moment of inertia of the shell-stiffener geometry
L	distance between the bulkheads
L_s	stiffener spacing
Mp'	plastic moment with axial force
n	number of circumferential lobes
p	external pressure acting
p_E	collapse pressure due to elastic buckling
p_{ISB}	interstiffener buckling collapse pressure
p_{OB}	overall buckling collapse pressure
p_{RC}	ring buckling collapse pressure
R	Mean radius of circular cylindrical shell
R_f	radius of ring at its neutral axis
t	shell thickness
t_f	flange thickness of stiffener
t_w	thickness of web
Y_s	centroidal distance of stiffener
β_0	initial non-dimensional imperfection
β	non-dimensional imperfection at a pressure, p
ν	Poisson's ratio
σ_{cr}	critical buckling stress
σ_E	elastic buckling stress
σ_y	yield stress of the material
σ_{yp}	yield stress of shell plating
σ_{ys}	yield stress of stiffener

2. Submarine pressure hull—inelastic buckling analysis

Submarine pressure hulls are in-common stiffened shell structures. Though the shell structures fail by elastic buckling, it is shown herein that for some shell-stiffener geometry and material properties they can fail inelastically. One such geometry of ring stiffened cylindrical shell has been considered for the study and is shown in Fig. 1.

Classical solutions are generally used for the analysis of stiffened cylindrical shells, in which three types of elastic buckling failures are mostly considered and there are, stiffener buckling, interstiffener buckling and overall buckling. The expressions for the collapse pressures of these failure modes are given by the well-known Bresse's equation, Von-Mises's equation and Bryant's equation, respectively (Ross, [6]), and are as follows.

Ring buckling collapse pressure:

$$p_{RC} = \frac{3EI_{xx}}{R_f^3 L_S} \tag{1}$$

Interstiffener buckling collapse pressure:

$$p_{ISB} = \frac{2.42E(t/2R)^{5/2}}{(1-(v)^2)^{0.75}[(L_S/2R) - 0.447(t/2R)^{1/2}]} \tag{2}$$

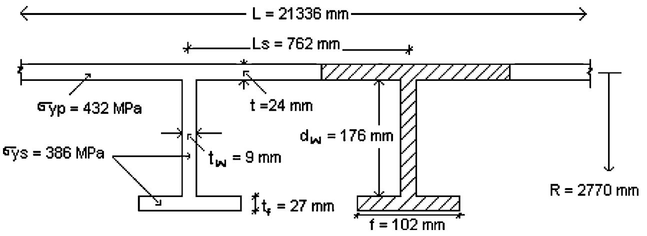

Fig. 1. Shell-stiffener geometry.

Overall buckling collapse pressure

$$p_{OB} = \frac{m^4Et}{R\left(n^2 - 1 + \frac{m^2}{2}\right)(n^2+m^2)} + \frac{EI_{xx}(n^2-1)}{R^3 L_S} \tag{3}$$

where

$$R_f = R - Y_S - \frac{t}{2} \text{ and } m = \frac{\pi R}{L} \tag{4}$$

The elastic buckling pressures obtained, by using the above classical expressions, for the considered submarine pressure hull are given in Table. 1.

In the present study of submarine pressure hull, the elastic stresses obtained are higher than the yield stress of the material considered. This indicates that the failure of the structure is due to inelastic buckling and hence it calls for a rigorous inelastic analysis. Three different approaches viz. 'Johnson–Ostenfeld inelastic correction', 'imperfection method' and 'finite element approach' have been considered for the determination of the collapse pressure when failure is governed by inelastic buckling.

2.1. Johnson–Ostenfeld inelastic correction

A structure, which has high elastic buckling strength, will normally buckle in the elasto-plastic range. Most design rules of classification societies approximately calculate the inelastic buckling strength of shell elements by a correction for plasticity applied to the elastic buckling strength, using the so-called

Table 1
Elastic buckling pressures

Sl. no	Type of buckling	Collapse pressure (MPa)	Stress in stiffener (MPa)
1	Ring buckling	4.90999	458.044
2	Interstiffener buckling	5.81423	542.399
3	Overall buckling	6.34461	591.878