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Buckling of plates with strengthenings

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Abstract

This paper studies the buckling behaviour of simply supported square plates, which have weakening or strengthening bands. The weakening/strengthening bands are equally spaced and their thickness is either decreased or increased. The analysis assumes that the stress state in the plate before and during the buckling process remains in the elastic range. Two cases of plate loading are studied, one with compressive forces and one with tangential forces. The buckling coefficients are calculated for different numbers and thicknesses of strengthening/weakening bands. The thickness of strengthening/weakening bands and the thickness of the remaining plate is varied so that the volume/weight of the plate remains constant. In one case it is found that the buckling load at constant weight of the plate can be increased by 118% if an optimal ratio of the thickness of strengthening bands and the thickness of the remaining plate is used.

Keywords: Square plate; Elastic buckling; Buckling coefficient; Buckling mode; Eigenvalue; Finite element method

1. Introduction

The paper treats elastic stability of homogeneous square plates, which are strengthened with equally spaced bands of increased plate thickness, $t>h$, or weakened if the plate thickness of these bands is decreased, $t<h$ (see Fig. 1). The plates are simply supported along all four edges. Two different cases are considered. In the first case the two edges, parallel to the y-axis, are loaded by an external uniformly distributed compressive in-plane force per unit length N_x (Fig. 1(a)) and the width of the strengthening/weakening bands is $a/11$. The direction of strengthening/weakening bands is parallel to the x-axis [5],[7]. In the second case all four edges are loaded with uniformly distributed tangential in-plane forces per unit length $N_{xy}=N_{yx}$ (Fig. 1(b)) and the width of the strengthening/weakening bands is $a\sqrt{2}/11$. The direction of strengthening/weakening bands in this case is parallel to the direction $y=x$ [6].

The aim of this paper is to study the influence of strengthening/weakening bands on the buckling load. Additionally, considering the condition of constant volume of the plate, the existence of an optimal t/h ratio at a given number of strengthening/weakening bands m, which maximizes the buckling load will be shown.

2. Solution of the buckling problem

The initial elastic stability of the plate with strengthening bands is analysed using the finite element method (FEM). Non-conformal triangular elements with nine degrees of freedom are adopted. It is known from [1] that for this element the approximate solution converges towards the exact one, if the finite element mesh is generated by means of three sets of equally spaced lines (see Fig. 2). In the study by Rubeša [2] a Fortran code, which used the described element, was presented. By modeling only one quarter of the plate, the code was used to solve some examples of axi-symmetric buckling with maximum 36 elements. In our case the problem is more demanding since axi-symmetric as well as asymmetric buckling modes are considered. The use of FEM for initial stability computations yields a generalised eigenvalue problem

$$(\mathbf{A}+\lambda\mathbf{B})\mathbf{q}=0, \tag{1}$$

where $\mathbf{A}$ is the global stiffness matrix used in linear theory of thin plates, $\mathbf{B}$ is the global matrix of geometric stiffness and $\mathbf{q}$ is the vector of node displacements and rotations. λ is the eigenvalue of the upper homogeneous system used for calculation of the buckling coefficient k and the buckling mode (eigenvector $\mathbf{q}$). The matrix $\mathbf{A}$ is symmetric and positively definite and, if the plate is stably supported, also

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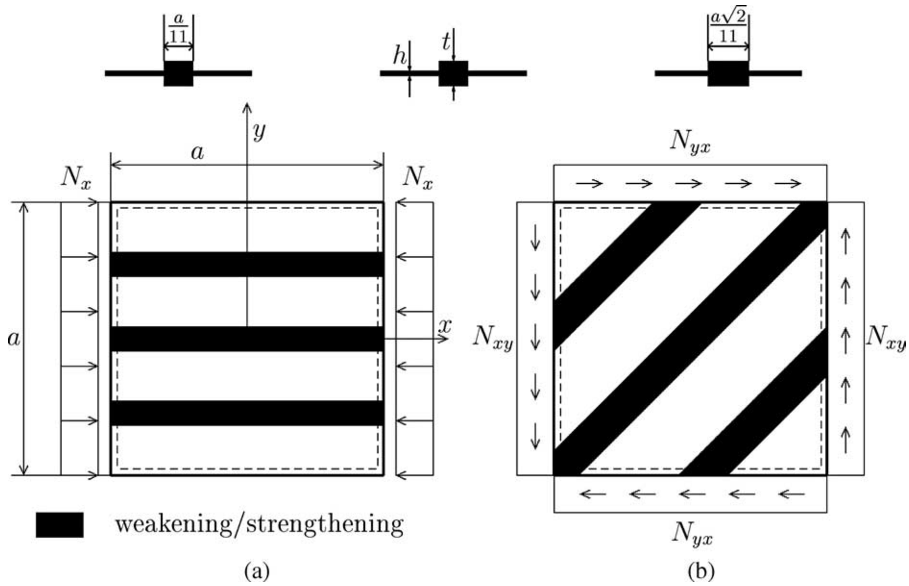

Fig. 1. Studied cases: (a) compressive forces N_x; (b) tangential forces N_{xy}.

non-singular. Using some matrix manipulations a usual eigenvalue problem can be obtained

$$(\mathbf{A}^{-1}\mathbf{B}-\omega\mathbf{I})\mathbf{q}=0, \tag{2}$$

where $\mathbf{I}$ is the unit matrix and $\omega=-\lambda^{-1}$. Since $\mathbf{A}$ is symmetric and positively definite, it is possible to perform a Cholesky decomposition [3] where $\mathbf{A}=\mathbf{L}\cdot\mathbf{L}^{\mathrm{T}}$ being $\mathbf{L}$ the lower triangular matrix. By multiplying Eq. (2) by $\mathbf{L}^{-1}$ from the left side a new equation is obtained

$$(\mathbf{L}^{-1}\mathbf{B}\mathbf{L}^{-\mathrm{T}}-\omega\mathbf{I})\mathbf{L}^{\mathrm{T}}\mathbf{q}=0. \tag{3}$$

Introducing a new matrix $\mathbf{C}=\mathbf{L}^{-1}\mathbf{B}\mathbf{L}^{-\mathrm{T}}$ and vector $\mathbf{y}=\mathbf{L}^{\mathrm{T}}\mathbf{q}$ the eigenvalue problem can be rewritten as

$$(\mathbf{C}-\omega\mathbf{I})\mathbf{y}=0. \tag{4}$$

For the solution of the initial stability problem, which is obtained by solving the eigenvalue problem Eqs. (2) or (4), only the maximum eigenvalue and eigenvector are needed and the power method [3] was used to calculate the solution. Assuming that the solution of the eigenvalue problem requires approximately the same number of computations as the Gauss method of solution of a linear system of equations, and on the basis of the round-up error effect on the precision of the solution, it is unreal to expect that at usual (single) precision of computing, the results were precise at any digit at all [3].

We solved the eigenvalue problem using single precision computation in two ways, firstly by solving Eq. (2) and secondly by solving Eq. (4). As expected, the results were different and we concluded that in terms of precision none of the methods had advantage. On the other hand, when using double precision computation, both methods gave equal results for the first eight digits. From this, we can conclude that the deviation of the calculated values from available analytical solutions is a result of insufficient finite element discretisation of the plate.

Considering the number of computing operations, the advantage is given to the conversion of the generalized eigenvalue problem into the usual eigenvalue problem by means of Cholesky decomposition of the matrix $\mathbf{A}$, Eq. (4). A different conclusion could perhaps be made, if the special structure of both global stiffness matrices, where both have most elements equal to zero, was considered.

3. Problem formulation

For a reference square plate with constant thickness h_0 and edge length a, the buckling stress can be written

$$\sigma_{\mathrm{cr}}=\frac{N_{\mathrm{cr}}}{h_0}=k\frac{\pi^2E}{12(1-\nu^2)}\left(\frac{h_0}{a}\right)^2, \tag{5}$$

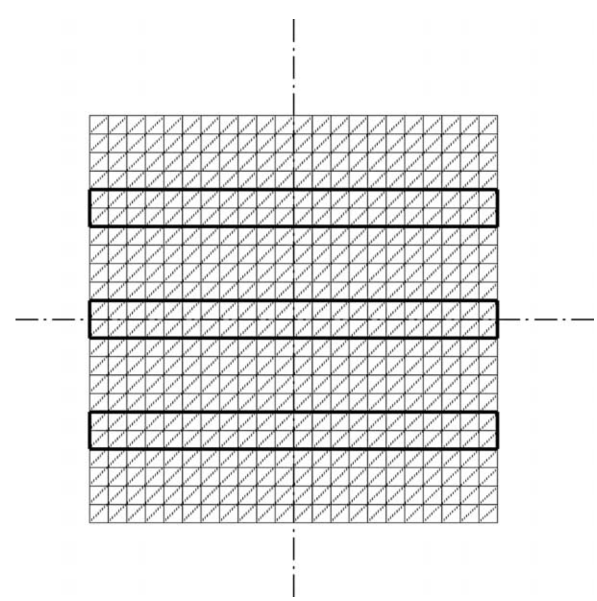

Fig. 2. Finite element model of the plate.