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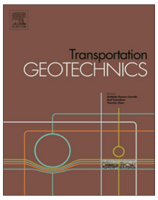

Railway critical velocity – Analytical prediction and analysis

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ABSTRACT

When high speed trains travel close to the wave propagation velocity of the supporting track-ground system, large amplitude track deflections are generated. This has safety implications, and also results in a significant increase in track maintenance due to subgrade deterioration. Thus, this paper presents a method to rapidly predict the speed at which these 'critical velocity' effects occur. The method is based upon a dispersion analysis of both the track (either ballast or non-ballasted/slab track) and the underlying ground, which are treated as uncoupled systems. Unlike previous approaches, the new calculation approach is fully automated thus not requiring any post-processing to extract the soil dispersion curve. It also works for soil layers of arbitrary depth, uses minimal computing power and can calculate critical speeds associated with higher soil modes. The dispersion based method can be deployed on new/existing lines via a drop-weight test, or using existing geotechnical data. Its accuracy is tested by comparing the results against an alternative semi-analytical, quasi-static railtrack model, and found to be 97% accurate. The code is useful for railway track infrastructure design and its short run times mean it can be used as a scoping tool for newly proposed high speed railroad lines. To obtain new insights into the key variables effecting critical velocity, a sensitivity analysis is undertaken using 1000 random soil profiles. It is found that on average, for the same track height, slab tracks are less likely to encounter critical velocity issues than ballasted tracks because their critical speed is typically 11% higher. It is also shown that track height plays an important role with increases in slab track thickness and reductions in ballasted track thickness both causing increases in critical velocity. Furthermore, it is found that soil saturation affects critical speed considerably (by up to 12–17% depending on track type) because changes to Poisson's ratio alter the dispersion characteristics of layered soils in the mid-frequency range, where critical velocity effects occur. Finally, it is shown that railpad stiffness has a low influence, and that increasing the rail bending stiffness on ballasted tracks increases critical speed.

Introduction

High speed rail is undergoing rapid growth, in part, due to increases in operational train speed. These elevated train speeds create new challenges for high speed rail infrastructure. One challenge is that of critical velocity, associated with the propagation of stress waves in the track and supporting soil. Critical velocity effects occur when the wavelengths of the waves in both the track and soil coincide. When this occurs, the low frequency energy generated due to the vehicle passage (e.g. carriage passage frequency), is magnified and the track structure is subject

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to large amplitude vibrations (Woldringh and New, 1999; Connolly et al., 2015; Wang et al., 2015; Connolly et al., 2014; Huang and Chrismer, 2013). These vibrations are a safety concern due to the increased risk of loss of contact between wheel and rail. Furthermore, they cause increased subgrade deterioration.

A frequently cited critical velocity case study is Ledsgard, Sweden, where shortly after the opening of the X2000 service, large vibrations were experienced in the track Takemiya, 2003; Kaynia et al., 2000; Madshus and Kaynia, 2000. The cause was attributed to a low shear wave velocity layer in the soil stratum, which was lower than the train speed. Temporarily, speed restrictions were enforced, however to solve the problem, dry deep soil mixing (at great cost) was used to strengthen the soil stratum (Holm et al., 2002).

If high profile, costly cases like Ledsgard, Sweden are to be avoided, it is vital that critical velocity effects can be predicted before construction. Therefore, in an attempt to understand the behaviour of a moving load, early approaches approximated the problem as a moving point load on an elastic half-space (Kenney, 1954; Fryba, 1972; Lamb, 1904). To tailor the problem for railway applications, and to investigate critical velocity effects, Krylov (1995) and Degrande and Lombaert (2001) used a numerical approach and approximated the movement of a train wheel as a moving point load. The track was modelled analytically and a Green's function used to calculate the soil response. Alternative formulations were analysed by Dieterman and Metrikine (1996) and Barros and Luco (1995).

Sheng et al. (2003) and Thompson (2009) also proposed an alternative semi-analytical approach where the track was modelled analytically, and the soil modelled using an integral transformation approach. Model results were benchmarked against track displacement recordings undertaken at Ledsgard, Sweden and found to produce reliable results. This approach was also extended to include poroelastic effects (Chahour et al., 2014).

In contrast to analytical modelling, Ferrara et al. (2013), Yang et al. (2009), and Nsabimana and Jung (2015) proposed 2D finite element (FE) models to analyse critical velocity. Full 3D approaches have also been used (El Kacimi et al., 2013; Connolly et al., 2013; Varandas et al., 2011; Galvin and Domínguez, 2007; Kouroussis et al., 2011; Hall, 2003; Shih et al., 2014) in both the time and frequency domains. An advantage of time domain modelling is that non-linear effects can be considered (Paixão et al., 2015), however simulation run times can be prohibitive. Therefore, to increase efficiency, 3D finite element models have been adapted to incorporate modal sub-structuring that reduces the overall number of degrees of freedom (Arlaud et al., 2014; Ferreira and López-Pita, 2015).

Furthermore, a variety of hybrid models have been proposed. For example, combined FE and boundary element (BE) methods (Galvin et al., 2010; Andersen et al., 2007; Colaço et al., 2015) have been investigated to eliminate the need for absorbing boundary conditions. Alternatively, the finite element has been combined with semi-analytical approaches (Triepaischajonsak and Thompson, 2015), the thin layer method (Bian et al., 2014), and also the discrete element method to analyse ballast settlement at critical speed (Huang and Chrismer, 2013).

To reduce computational requirements, the track-soil modelling formulation can be reduced to 2.5D, where the track is assumed invariant in the direction of train passage (Alves Costa et al., 2012). Alves Costa et al. (2010) used this technique, combined with a linear equivalent formulation, to investigate critical velocity effects. Strong agreement was found with the results from Ledsgard, Sweden, and it was also concluded soil that non-linearity plays an important role in critical velocity phenomena.

When investigating potential critical velocity effects on a newly proposed line, two key quantities that designers need to know are the critical speed value(s), and also the dynamic amplification that will occur at different speeds (Connolly et al., 2015). This paper focuses on the calculation of the critical speed value. For a moving point load on the surface on a homogenous half-space, it is relatively straightforward to compare the load speed and natural soil wave speed. In contrast, for a railway track resting on a multi-layered soil, the problem is more complex. In this situation, the speed of propagation of surface waves within the track and soil systems is frequency dependent, defined as velocity dispersion.

Therefore, the dispersion characteristics of the ground have been analysed in an attempt to approximate critical speed values (Sheng et al., 2003; Sheng, 2001; Bian et al., 2014; Zhai et al., 2015). In most cases, the soil dispersion curves are compared with moving load curves and have generated reasonably good results. Building upon this approach, Alves Costa et al. (2014) investigated the relationship between the dispersion characteristics of both the track and soil. It was found that the point where the velocities in both systems for a given wavelength were equal gave an accurate approximation of the critical speed value. In this formulation, the track dispersion curve was computed analytically, and the soil modelled using the Thompson–Haskell approach.

Although the Thompson–Haskell method is an efficient approach for soil dispersion curve calculation, it is not well suited for certain stratum configurations, such as thick soil layers. Furthermore, the integral approach usually results in a 2D spectrum which must be post-processed to extract the true dispersion curve. To avoid these drawbacks, this work builds upon the work of Alves Costa et al. (2014) and presents a fully analytical approach for both soil and track dispersion curves. Therefore the critical velocity value can be computed entirely automatically, which is attractive for critical velocity scoping exercises. Furthermore, the approach works for large soil layers and has minimal run times. Due to these minimal run times, a sensitivity analysis is also undertaken to investigate the effect of ballast height, slab track characteristics and soil layering approximations on critical velocity.

Numerical modelling

Dispersion analysis

The term dispersion relates the speed and frequency of wave propagation within a structure. In the context of a