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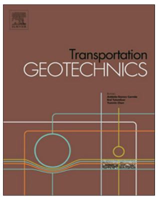

Critical speed of railway tracks. Detailed and simplified approaches

Pedro Alves Costa [*], Aires Colaço, Rui Calçada, António Silva Cardoso

Faculty of Engineering of the University of Porto, R. Roberto Frias, Porto, Portugal

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ABSTRACT

The dynamic amplification effects of the response due to a moving load on the surface of an elastic solid has been object of research for more than a century. However, if in the beginning of the last century the problem had only theoretical interest, this is no longer true. Indeed, the recent advancements in the rolling stock, which can now reach speeds higher than 500 km/h, brought this kind of problems to the engineering practice, mainly to high speed railway engineering. The present paper approaches this problem focusing on railway engineering. The departing point is the theoretical formulation of the critical speed problem of a moving load on the surface of an elastic solid. From the usage of 2.5D detailed models it was possible to understand the influence of the embankment and track properties on the critical speed. However, to avoid complex numerical models, which are very demanding from the computational point of view, a simplified approach is proposed for the computation of the critical speed of track–embankment–ground systems. The results of the simplified approach are compared to those achieved by detailed methods, also presented in this paper, and the proposed expedite methodology is found to be very accurate.

Introduction

Environmental concerns and the demand for efficient and rapid transportation systems have lead to an impressive development of railway systems during the last decade. In order to increase the economic viability of this transportation system, the capacity of the infrastructure has to be improved and this leads to a paramount need of increase of train speed and axle weight. Responding to these demands, the operational speed of trains has increased over the last years: in France the TGV operates at a speed of 320 km/h, the Chinese railway administration expects to increase high speed traffic up to 400 km/h in the near future and the operational speed expected for the HS2 project in the UK reaches 420 km/h. Moreover, taking the recent developments of the rolling stock technology, which allowed the French TGV to reach a record speed of 574.8 km/h, it is easy to predict that even faster trains will be introduced in the near future.

The introduction of high-speed systems in the railway network across the world has brought new problems to railway geotechnics (among others), namely because of the significant amplification of train-track vibrations at high speeds (Kaynia et al., 2000; Madshus et al., 2004; Woldringh and New, 1999; Krylov, 1995), which can compromise the safety and stability of the infrastructure. Actually, the dynamic amplification effects of the response due to a moving load on the surface of an elastic solid has been object of research for more than a century (Eason, 1965; Lamb, 1904; Kenney, 1954; Dieterman and Metrikine, 1996, 1997; Barros and Luco, 1994, 1995). However, if in the beginning of the last century the problem had only

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[*] Corresponding author.

E-mail addresses: pacosta@fe.up.pt (P.A. Costa), aires@fe.up.pt (A. Colaço), ruiabc@fe.up.pt (R. Calçada), scardoso@fe.up.pt (A.S. Cardoso).

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theoretical interest, this is no longer true. The increase of train speed together with the demand of crossing alluvionar soft soils can bring this problem to engineering practice, as was already observed in Ledsgard's famous case study where train speed approaches the critical speed of the track–ground system (Hall, 2003; Alves Costa et al., 2010a; Madshus and Kaynia, 2000). By definition, critical speed is the velocity of the moving non-oscillating load that conducts to the higher amplification of the dynamic response, i.e., the speed of the load that gives rise to a resonant like problem. The critical speed is fully dominated by the properties of wave propagation in the embankment-ground system and of the bending wave propagation in the track, as previously discussed by Dieterman and Metrikine (1996, 1997) and Sheng et al. (2004), among others.

Due to the practical interest of the problem, considerable research effort has been made during the latter decade, during which several analytical and numerical approaches have been proposed for the assessment of critical speed. The problem is quite complex due to the 3D characteristic of the system and to the moving character of the load. Among other studies, special reference is given to the semi-analytical model proposed by Sheng et al. (2004), where the ground is simulated by an analytical integral transformed approach (Haskell-Thompson procedure) and the track is modelled by a range of beams, masses and spring-dashpots in order to take into account the dynamic behaviour of distinct components of the track, namely: rails, railpads, sleepers and ballast layer. However, when the geometry of the system is not compatible with the restrictions imposed by a semi-analytical approach, namely when the embankment height is not negligible, several authors support the resource to fully 3D numerical methods, as for instance the BEM (Andersen and Nielsen, 2003; Adam et al., 2000) or the FEM (Hall, 2003; Connolly et al., 2013; Woodward et al., 2013; Kouroussis et al., 2009; El Kacimi et al., 2013). In spite of the powerful capacities of these numerical models, computational demands hinder their intensive usage on engineering practice. To avoid this drawback, 2.5D formulations, applicable to both FEM and BEM, are the chosen method in several situations (Alves Costa et al., 2010a,b, 2012; Yang and Hung, 2001; Galvín et al., 2010), since the assumption of invariant properties and geometry of the domain along the track development direction enables the discovery of an alternative procedure where only the cross-section of the problem needs to be discretized. Even though computational effort introduced by the 2.5D approach is highly reduced, computational time remains considerable. Moreover, it should be noticed that the latter methods are not available in commercial codes, difficulting the transfer of knowledge from the academia to engineering practice.

So, the physical phenomenon concerned to the critical speed assessment is well established from a theoretical point of view, being available numerical tools to deal with the problem. However, several aspects should be clarified if we wish to focus on the practice, namely: (i) there is a demand for simplified methods that allow easier and faster computation, to determine the critical speed; (ii) there is a demand for better knowledge of how geotechnical conditions, including soil non-linear behaviour, can influence the problem. In other words, a bridge should be built between physical concepts (using the wave propagation theory) and practical geotechnical railway engineering. As these demands are ambitious, the present paper aims to contribute to the first one, the development of simplified methods.

In the following sections, the critical speed problem is revisited from the background theory of wave propagation in a half-space in order to assess the dynamic response of the track–ground system. The influence of geotechnical layering as well as of the properties of the track on critical speed of the system is analysed. In spite of the intensive usage of advanced numerical modelling strategies, namely the 2.5D FEM-BEM approach, the present paper investigates the accuracy of simplified methods. A simplified method, inspired in a previous research performed by Sheng et al. (2004), is extended and validated allowing to predict the critical speed of the track–embankment–ground system. Contrarily to the detailed numerical methods, the simplified approach allows for the assessment of the critical speed of the system in few seconds with considerable accuracy and avoiding undesirable numerical complexity

Finally, the paper ends with the main remarks pointed out through the study developed.

Problem outline – moving loads applied directly on the ground surface

The solution to the problem of dynamic amplification of the response of an elastic solid due to traffic loads was found several years ago. However, due to its didactic value, this problem is here revisited regarding different geotechnical conditions, as depicted in Fig. 1. The first scenario comprises a homogenous half-space, while the remaining two take ground layering into account. The second scenario assumes the presence of an upper softer layer. This situation is inverted in the third scenario.

An unitary vertical load is assumed, spread over an area of $2a \times 2a = 2 \times 2\ \mathrm{m}^2$. The loading, moving along x direction with a c speed and being its centre for t = 0 s at the referential origin, can be mathematically described by:

$$p_z(x,y,0,t) = \left\{ \begin{array}{c} \frac{1}{2a \times 2a}\delta(x-ct)|x| < a, |y| < a \\ 0 \end{array} \right\} \tag{1}$$

Analysing the problem in light of the wave propagation theory, it is helpful to transform the loading to the wavenumber-frequency domain (where k_1 is the Fourier image of x and ω is the Fourier image of time):

$$p_z(k_1,y,0,\omega) = \left\{ \begin{array}{c} \frac{1}{2a}\frac{\sin(k_1 a)}{k_1 a}\delta(\omega - k_1 c)|y| < a \\ 0 \end{array} \right\} \tag{2}$$

This means that expression (2) has only physical meaning when $w = k_1 c$ corresponds to a line in the frequency-wavenumber space.

Starting with the scenario 1, the adimensional vertical displacements of the ground surface are depicted in