Transportation Research Part A 44 (2010) 39–51

Contents lists available at ScienceDirect

Transportation Research Part A

journal homepage: www.elsevier.com/locate/tra

Existence of self-financing and Pareto-improving congestion pricing: Impact of value of time distribution

Yu (Marco) Nie *, Yang Liu

Department of Civil and Environmental Engineering, Northwestern University, 2145 Sheridan Road, Evanston, IL 60208, United States

ARTICLE INFO

Article history:
Received 18 December 2008
Received in revised form 14 September 2009
Accepted 23 September 2009

Keywords:
Value of time
Self-financing
Pareto-improving
Congestion pricing
Log-normal

ABSTRACT

This paper considers a static congestion pricing model in which travelers select a mode from either, driving on highway or taking public transit, to minimize a combination of travel time, operating cost and toll. The focus is to examine how travelers' value of time (VOT), which is continuously distributed in a population, affects the existence of a pricing-refunding scheme that is both self-financing (i.e. requiring no external subsidy) and Pareto-improving (i.e. reducing system travel time while making nobody worse off). A condition that insures the existence of a self-financing and Pareto-improving (SFPI) toll scheme is derived. Our derivation reveals that the toll authority can select a proper SFPI scheme to distribute the benefits from congestion pricing through a credit-based pricing scheme. Under mild assumptions, we prove that an SFPI toll *always* exists for concave VOT functions, of which the linear function corresponding to the uniform distribution is a special case. Existence conditions are also established for a class of rational functions. These results can be used to analyze more realistic VOT distributions such as log-normal distribution. A useful implication of our analysis is that the existence of an SFPI scheme is *not* guaranteed for general functional forms. Thus, external subsidies may be required to ensure Pareto-improving, even if policy-makers are willing to return all toll revenues to road users.

1. Introduction

The study of congestion pricing is traced back to the seminal work of Pigou (1920) and Knight (1924). From then on, the underlying economic model has been developed (e.g. Beckmann et al., 1956; Walters, 1961; Vickrey, 1963, 1969). The essential idea behind congestion pricing is to charge road users a fee corresponding to the cost of congestion they cause. This charge, as a price signal, dampens travel demand (hence alleviate traffic congestion) by "taxing out" motorists whose benefits from making their trips on the "tolled" road do not justify the payment of the toll. Road pricing in various forms has been practiced in a number of cities (e.g. Small and Gomez-Ibanez, 1998; Nakamura and Kockelman, 2002; May and Sumalee, 2003; Eliassona and Mattsson, 2006; Eliassona et al., 2009), and existing empirical evidence (e.g. Button, 1986; Small, 1993) indicates that pricing a road does reduce demand for its use and consequently alleviate congestion.

Despite congestion pricing is theoretically appealing and becoming increasingly easy to implement, the public appears reluctant to embrace this policy. Among many arguments against road pricing, the equity issue is frequently cited. Marginal cost pricing, for example, is known to produce higher user cost and lead to inequities among users (see e.g. Yang and Zhang, 2002; Hau, 2005). It has been argued that congestion pricing is easier to draw support from the public if it is Pareto-improving, that is, alleviating traffic congestion while increasing nobody's travel cost (Lawphongpanich et al., 2004; Hau, 2005). Lawphongpanich and Yin (2008) proposed a network model that includes the above Pareto-improving requirement as

* Corresponding author. Tel.: +1 847 467 0502.
E-mail address: y-nie@northwestern.edu (Yu (Marco) Nie).

0965-8564/$ - see front matter
doi:10.1016/j.tra.2009.09.004

constraints, but ignores the heterogeneity in the value of time. On the other hand, many (e.g. Evans, 1992; Arnott et al., 1994; Hau, 1998) observed that if toll revenues are not refunded, road pricing is *regressive* in that its benefit rises with income. Namely, because those with higher incomes usually have higher value of time (VOT), they benefit more from congestion relief promised by road pricing. To resolve this equity problem calls for redistributing toll revenues among users. This paper is focused on the revenue redistribution schemes and the role the VOT distribution plays in these schemes.

Small (1992) showed that congestion pricing may be progressive if a lump sum refunding is implemented, i.e. an equal travel allowance for all commuters. Similar schemes have been examined by others, see e.g. Goodwin (1989), Poole (1992) and DeCorla-Souza (1995). Recently, Kockelman and Kalmanje (2005) proposed a credit-based congestion pricing strategy, in which all drivers receive a monetary travel allowance (as a portion of toll revenues) to use on the roads. The strategy subsidizes those who would like to cash out the credit by staying off the road, with toll revenues collected from those who are willing to pay out-of-pocket for using the road.

Redistribution schemes have also been studied using various mathematical models. Adler and Cetin (2001) proposed a redistribution model based on a two-route bottleneck model (Arnott et al., 1990), in which toll revenues collected from users on a more desirable route are used to compensate users on a less desirable route. Without considering heterogeneity of users, they showed that the approach will eliminate queuing time and reduce the travel cost of all users. For a single Origin–Destination (O–D) pair connected by a number of parallel routes, Eliasson (2001) showed that a tolling and refunding system that reduces system travel time and refunds the toll revenues equally to all users will make everyone better off, regardless of traffic flow model or VOT distribution. Similar results were obtained in Yang and Guo (2005), which concluded that a congestion pricing scheme that reduces system travel time and redistributes the toll revenues to all users is always Pareto-improving. Heterogeneous VOT is considered using a multiple-class network equilibrium model in Yang and Guo (2005). Arnott et al. (1994) studied the welfare effects of congestion tolls using Vickery's model (Vickrey, 1969), in which commuters differ from each other not only in VOT, but also in their value of late/early arrival costs as well as desirable arrival time. Arnott et al. (1994) found that Pareto-improving is not guaranteed with an equal lump sum return. In particular, drivers with low VOTs could remain worse off if they are minority. Recently, Liu et al. (2008) examined the existence of Pareto-improving and revenue-neutral pricing schemes in a bi-modal network. The model charges a toll on a more desirable mode (automobile) and subsidizes the less desirable one (transit) using the revenues. With a general VOT distribution of commuters, an anonymous toll scheme is sought to: (1) reduce system travel time, (2) ensure no individual user to be worse off, and (3) guarantee revenue-neutral (all toll revenues are redistributed). Based on a static user equilibrium model, a general condition is proposed to ensure the existence of a Pareto-improving scheme when all toll revenues are returned to users. Liu et al. (2008) also showed that the condition is guaranteed for uniformly distributed VOT functions.

It is well-known that actual VOT distributions are not uniform. For example, Ben-Akiva et al. (1993) suggested that VOT in a population follow a log-normal distribution. Is the general condition given in Liu et al. (2008) satisfied for a more general VOT distribution? In other words, if all toll revenues are to be returned to road users, is it always possible to find a toll scheme to alleviate congestion (i.e. reduce system travel time) while making nobody worse off? The present paper aims to address precisely this question.

A main finding of the paper is that whether Liu et al.'s condition is satisfied highly depends on the shape of the VOT distribution. We prove that any concave VOT distribution function satisfies the condition, and develop existence conditions for a class of first-order rational distribution functions. These distributions are useful because they could provide better approximations to realistic VOT distributions such as log-normal (see Section 4.3 for details). Nonetheless, general VOT distributions do not guarantee Liu et al.'s condition, which implies that *an equal lump sum return does not always guarantee Pareto-improving without external subsidy*. This result is interesting and, apparently, disagrees with the previous results such as Eliasson (2001) and Yang and Guo (2005). We note that the reason for the discrepancy is that our model imposes a mode-specific *operating cost* (see Section 2 for details).

We also provide an alternative derivation of Liu et al.'s existence condition. The derivation relaxes the requirement for revenue neutral and thereby reveals that a Pareto-improving toll scheme can generate positive net revenues. In turn, this implies that a toll authority could withhold all or a portion of these net revenues without making anybody worse off. To be more specific (see Section 3 for details), a Pareto-improving toll scheme can be either revenue-neutral (i.e. returning all net revenues to users, which is considered in Liu et al. (2008)) or revenue-maximization (i.e. the toll authority withholds all net revenues). To some extent this is a good news because it gives policy-makers flexibility to utilize toll revenues for other investments (such as the road construction and maintenance), without undermining the feasibility of the toll scheme. It follows that a self-financing and Pareto-improving (SFPI) toll scheme is unique only when the net revenue is zero.

This paper is organized as follows. The next section reviews the bi-modal pricing model and its user equilibrium solution. In Section 3, a condition that guarantees the existence of an SFPI toll scheme is derived and its relationship with the one proposed in Liu et al. (2008) is discussed. In Section 4, the existence of an SFPI toll is considered for several special VOT functions, including concave, linear and a class of rational functions. We present numerical results in Sections 5 and 6 concludes the study.

2. The bi-modal pricing model and its solutions

Consider a network with one O–D pair, which has a fixed demand d and is connected by two routes: a highway and a transit line. Denote the travel time on the highway by $\tau(f)$, a strictly increasing and convex function of the number of people