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A theory and implications on dynamic marginal cost

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Abstract

This paper extends the conventional static marginal cost analysis to the dynamic one based on the time-dependent queueing analysis at a bottleneck. First, the supply function is reformulated so as to incorporate dynamically congestion phenomena. And, the marginal cost is shown to be more closely related to the duration of congestion rather than the personal cost, since a slight change of demand at one time affects an entire traffic condition thereafter. Next, the analysis is extended so as to include the departure time choice using previous departure time choice theory. Several implications such as road pricing schemes and ramp control strategies are also discussed.

Keywords: Marginal cost; Bottleneck; Departure time; Queueing delay; Schedule delay

1. Introduction

This study extends the static marginal cost analysis to the dynamic one so that the dynamic bottleneck phenomena are properly taken into account in its supply curve. Conventionally, the demand–supply equilibrium analysis has been extensively studied under the static framework and the marginal cost pricing strategy was proposed so as to maximize user surplus (=consumer's surplus). However, in its supply function, the static analysis has not well considered the time-dependent queue evolution, which causes dominant delay.

On the other hand, in traffic engineering field, although time-dependent queueing analysis has been commonly employed, the elasticity of demand has seldom been dealt with. In other words, impacts of supply performance on the demand have not been well discussed, although time-dependent supply performance such as queue evolution has been dynamically studied and modelled given fixed demand.

This study hence attempts to extend the conventional static demand–supply analysis to one in the dynamic framework by focusing on time-dependent queue evolution at a bottleneck. First, after a quick review of the conventional static analysis, the dynamic marginal cost at a bottleneck is formulated and the optimality condition is shown so as to maximize the user surplus based on the queueing theory. In the conventional static analysis, the cost associated with a travel is defined as a function of demand. However, at a bottleneck,

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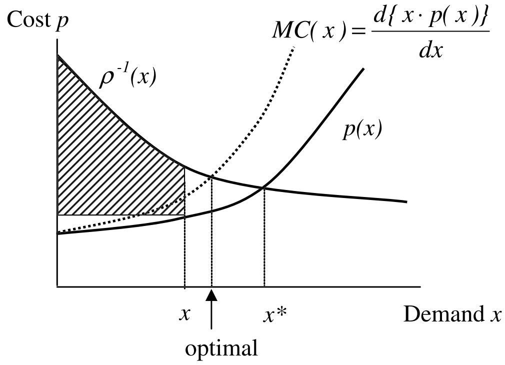

Fig. 1. Static demand–supply equilibrium.

when the demand exceeds its capacity, a queue would form and the cost due to the queue is no longer described only by the static demand but depends upon the time-dependent history of the demand profile. Therefore, the supply function is here defined time-dependently so as to properly consider the dominant queueing delay.

Second, the analysis is extended to the one with departure time choice. The reason for this extension is that, for the short-term decision, travellers normally decide when to travel before giving up the travel. (In this sense, inclusion of route as well as mode choices would be possible future extension in the analysis of this category.) Given a total demand for an entire study period, time-dependent personal as well as marginal costs are derived under the elastic demand with departure time choice. For this extension, an existing theory on departure time choice with time constraints at destinations is employed (see Vickrey, 1969; Hurdle, 1981; Hendrickson and Kocur, 1981; Fargier, 1981; Smith, 1984; Daganzo, 1985; Kuwahara and Newell, 1987; Kuwahara, 1990).

2. Review of static marginal cost analysis

Let us briefly review the derivation of the marginal cost in the static framework and show that the marginal cost pricing maximizes user surplus (=consumer's surplus). The demand and supply functions are first defined as follows:

$$\rho(p) = \text{demand generated at personal cost } p, \tag{1}$$

$$p(x) = \text{personal cost when demand is } x. \tag{2}$$

The inverse demand function, $\rho^{-1}(x)$, and cost $p(x)$ are illustrated in Fig. 1 and the intersection of two curves is the equilibrium point at demand x^*. When the demand is x, the user surplus shown as the shaded area in Fig. 1 is written as[1]

$$F = \int_0^x \rho^{-1}(y)\mathrm{d}y - x \cdot p(x). \tag{3}$$

Taking the derivative with respect to demand x and setting it equal to zero yields

$$\rho^{-1}(x) = \frac{\mathrm{d}}{\mathrm{d}x}\{x \cdot p(x)\} = \mathrm{MC}(x). \tag{4}$$

This means that the marginal cost $\mathrm{MC}(x)$, which is the derivative of total cost with respect to demand x, must be equal to the inverse of demand function $\rho^{-1}(x)$ when the user surplus is maximized. Therefore, the intersection of $\mathrm{MC}(x)$ and the inverse demand function $\rho^{-1}(x)$ gives the optimal point which maximizes the user surplus.

[1] Consumer's surplus is generally defined as the difference between the maximum amount that the consumer would pay and the amount he or she actually pays (Mansfield, 1982). In addition, for the simplicity, we here consider only users' trip costs but excluding several other costs associated with the trip such as maintenance cost, environmental cost, etc.