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Reliability analysis of roof wedges and rockbolt forces in tunnels

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ABSTRACT

The ambiguous nature of the factor of safety is first discussed in the context of a symmetric roof wedge of a circular tunnel, where two different definitions of the factor of safety are shown to be reconcilable when using the reliability index computed with the first-order reliability method (FORM). The probabilities of failure based on the second-order reliability method (SORM) are also obtained for comparison with those of FORM and Monte Carlo simulations. The FORM and SORM analyses are then applied to a circular tunnel supported with elastic rockbolts in a homogeneous and isotropic elasto-plastic ground with the Coulomb failure criterion. The similarities and differences between the ratios of mean values to design-point values, on the one hand, and the partial factors of limit state design, on the other hand, are discussed. Finally, all this is used to show how a reliability-based design can be performed to obtain the length and spacing of rockbolts for a target reliability index.

1. Introduction

The factor of safety design approach has long been used by geotechnical engineers. More recent alternatives are the load and resistance factor design (LRFD) approach in North America, and the characteristic values and partial factors used in the limit state design approach in Eurocode 7. Yet another approach can play at least a useful complementary role to LRFD and Eurocode 7, namely the design based on a target reliability index that explicitly reflects the uncertainty of the parameters and their correlation structure. Among the various versions of reliability indices, that based on the first-order reliability method (FORM) for correlated nonnormals is most consistent. A special case of FORM is the (earlier) Hasofer–Lind index (1974) for correlated normal random variables. These reliability methods are described in Ditlevsen (1981), Ang and Tang (1984), Madsen et al. (1986), Haldar and Mahadevan (1999), Melchers (1999), Baecher and Christian (2003), for example.

In many geotechnical problems the limit state surface (LSS, which separates safe combinations of parametric values from unsafe combinations) is practically plane so that the probability of failure inferred from FORM based on the hyperplane assumption is sufficiently accurate. Nevertheless, the FORM results can be easily extended to the second-order reliability method (SORM) which accounts for the curvature of the LSS near the design point.

This study will first examine two existing definitions of factor of safety for a roof wedge in a tunnel, and then show how the ambiguities can be resolved and the very different factors of safety reconciled when one computes not the factor of safety but the FORM reliability index β. This is followed by a brief summary of two relatively intuitive and transparent FORM computational approaches of Low and Tang (2004, 2007), aiming at overcoming the language and conceptual barriers (Whitman, 1984) surrounding reliability analysis. The FORM procedure is then applied to compute the reliability index β of the roof wedge of a circular tunnel. Probabilities of failure are also obtained from SORM and Monte Carlo simulations, for comparison with those from FORM. All this forms the basis for introducing reliability based design of tunnel supports, specifically for rockbolting. For this purpose, the Bobet and Einstein (2011) deterministic formulations of tunnels reinforced with rockbolts are summarized. This deterministic set-up is then extended probabilistically to FORM and SORM reliability analyses of rockbolt force, and reliability-based design of the length and spacing of rockbolts for a target reliability index. In FORM the design point is a point on the boundary (the limit state surface) which separates safe combinations of parametric values (e.g., the mean-value point) from the unsafe combinations of parametric values. The design point is the most-probable failure combination of parametric values. The similarities and differences between the ratios of mean values to design-point values and the partial factors of limit state design are discussed.

This paper deals only with certain aspects of reliability, namely methodology and concepts, and not reliability in its broadest sense.

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2. A tale of two factors of safety, and reconciliation via FORM

A symmetric roof wedge of central height h and apical angle 2α in a circular tunnel of radius R is shown in Fig. 1. An analytical approach for assessing the stability can be based on Bray's (1977) two-stage relaxation procedure, as described, for example, in Sofianos et al. (1999) and Brady and Brown (2006). The first stage computes the confining lateral force H_0 on the wedge from the stress field and the geometries of the wedge and tunnel, for an assumed homogeneous, isotropic, linearly elastic, weightless medium. The second stage then assumes deformable joints and a rigid rock mass, to arrive at the normal force N acting on each joint surface.

The following kinematic condition is necessary for the formation of the symmetric roof wedge:

$$\alpha \leqslant \sin^{-1}\left[\frac{1}{1+h/R}\right] \tag{1}$$

Equality in the above equation means that the two joints are just tangent to the tunnel. For semi-apical angles α larger than the limit in Eq. (1), the two joints cannot intersect the tunnel, and no wedge of the type in Fig. 1 is formed.

Two different definitions of the factor of safety against wedge falling have been reported in the literature, each with its own rationale. The first appeared in Sofianos et al. (1999) and Brady and Brown (2006), for example. It is the ratio of the pull-out resistance of the wedge to the weight of the wedge, and was expressed as follows:

$$FS_1 = \frac{2MH_0}{W} \tag{2}$$

$$\text{where } M = \frac{\lfloor \cos^2\alpha\cos i(k_s/k_n) + \sin(\alpha - i)\sin\alpha \rfloor \sin(\phi-\alpha)}{[\cos\alpha\cos\phi(k_s/k_n) + \sin\phi\sin(\alpha-i)/\cos i]\cos i} \tag{2a}$$

$$H_0 = \frac{1}{2}pR[(1+K_0)C_{H1} - (1-K_0)C_{H2}] \tag{2b}$$

$$C_{H1} = \left(\frac{h}{R}+1\right) - \frac{1}{((h/R)+1)} \tag{2c}$$

$$C_{H2} = \left(\frac{h}{R}+1\right) - \frac{1}{((h/R)+1)^3} \tag{2d}$$

$$W = \gamma R^2[\cos^2\theta(\tan\theta + \cot\alpha) - \pi/2 + \theta] \tag{2e}$$

$$\theta = \cos^{-1}\left[\left(\frac{h}{R}+1\right)\sin\alpha\right] + \alpha \tag{2f}$$

In the above equations, W is the weight of the wedge (Fig. 1), α the semi-apical angle of the wedge, ϕ and i the effective friction and dilation angles of the joints, k_s and k_n the shear and normal stiffnesses of the joints, R the radius of the tunnel, p and K_0 the vertical in situ stress and the coefficient of horizontal in situ stress, h the clear height of the wedge (measured from the tunnel crown), γ the unit weight of the rock, and θ is the angle denoted in Fig. 1.

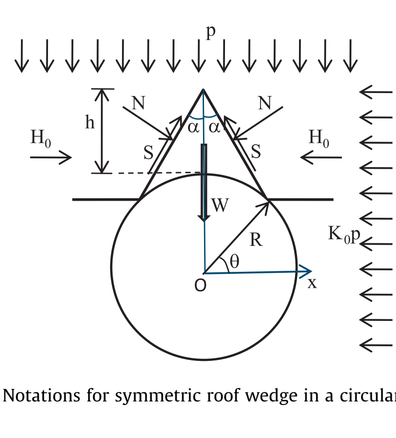

Fig. 1. Notations for symmetric roof wedge in a circular tunnel.

The second definition is similar in principle to that which has been long and widely used in soil and rock slope stability analysis, in the *Unwedge* program of *Rocscience.com*, and in Asadollahi and Tonon (2010), for example. It is the ratio of the available shear strength to the shear strength required for equilibrium. In the present context of tunnel roof wedge, this definition was given in Asadollahi and Tonon (2010) as follows (assuming the dilation angle of the joints $i = 0$):

$$FS_2 = \frac{2S\cos\alpha}{2N\sin\alpha + W} \tag{3}$$

where N and S are the normal and shear forces, calculated from the following equations (e.g., Brady and Brown, 2006), assuming $i = 0$:

$$N = \frac{H_0(k_s\cos^2\alpha + k_n\sin^2\alpha)\cos\phi}{k_s\cos\alpha\cos\phi + k_n\sin\alpha\sin\phi} \tag{3a}$$

$$S = \frac{H_0(k_s\cos^2\alpha + k_n\sin^2\alpha)\sin\phi}{k_s\cos\alpha\cos\phi + k_n\sin\alpha\sin\phi} \tag{3b}$$

The two definitions, Eqs. (2) and (3), are recast below – to facilitate understanding – in terms of N, W, α and ϕ. For simplicity, dilation angle i is assumed to be zero. If desired, a tunnel supporting force can easily be incorporated.

$$FS_1 = \frac{\text{Limiting wedge weight}}{\text{Actual wedge weight}} = \frac{2N\tan\phi\cos\alpha - 2N\sin\alpha}{W} = \frac{\tan\phi/\tan\alpha - 1}{W/(2N\sin\alpha)} \tag{4}$$

$$FS_2 = \frac{\text{Maximum available resisting forces}}{\text{Downward driving forces}} = \frac{2N\tan\phi\cos\alpha}{2N\sin\alpha + W} = \frac{\tan\phi/\tan\alpha}{1 + W/(2N\sin\alpha)} \tag{5}$$

where W is computed from geometrical relationships, Eqs. (2e) and (2f), and N from Eqs. (3a), (2b), (2c), (2d).

The "Limiting wedge weight" in Eq. (4) means the wedge weight at limiting equilibrium, i.e., the wedge weight that just causes failure. It is negative if $\phi < \alpha$.

For the case with dilation angle $i = 0$, the same FS_1 is obtained whether computed from Eq. (2) or Eq. (4), and the same FS_2 is obtained whether computed from Eq. (3) or Eq. (5). Nevertheless, the rationales, similarities and differences between FS_1 and FS_2 are rendered much more transparent in Eqs. (4) and (5) than in Eqs. (2) and (3). That FS_1 can be negative when $\phi < \alpha$ is also readily appreciated from Eq. (4). One may note that FS_1 by Eq. (4) – which is mathematically equivalent to Eq. (2) – can be very large and positive if W/N is small and $\phi > \alpha$, and negative if $\phi < \alpha$.

The two different definitions can give very different FS values, as shown in Table 1 for two example cases. The ratios of the FS_1/FS_2 for the two cases are 11.8/1.4 = 8.4, and 13.5/1.23 = 11.0.

Dividing Eq. (4) by Eq. (5), one obtains the ratio of the two factors of safety as:

Table 1
Computed FS_1 and FS_2, with p = 1 MPa, R = 2 m, k_s/k_n = 0.1, and γ = 0.027 MN/m^3.

	K_0	h/R	α (°)	ϕ (°)	W (MN)	N (MN)	FS_1	FS_2
Case 1	1	1	30	40	0.0740	1.926	11.8	1.40
Case 2	1.5	0.85	32	38	0.0573	2.913	13.5	1.23