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Describing failure in geomaterials using second-order work approach

François Nicot [a,*], Félix Darve [b]

[a] Institut de Recherche en Sciences et Technologies de l'Environnement et de l'Agriculture (IRSTEA), Erosion Torrentielle Neige et Avalanches (ETNA)-Geomechanics Group, Grenoble 38402, France

[b] Université Joseph Fourier (UJF)-Institut National Polytechnique de Grenoble (INPG)-Centre National de la Recherche Scientifique (CNRS), Laboratoire Sols, Solides, Structures et Risques, Grenoble 38041, France

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Abstract

Geomaterials are known to be non-associated materials. Granular soils therefore exhibit a variety of failure modes, with diffuse or localized kinematical patterns. In fact, the notion of failure itself can be confusing with regard to granular soils, because it is not associated with an obvious phenomenology. In this study, we built a proper framework, using the second-order work theory, to describe some failure modes in geomaterials based on energy conservation. The occurrence of failure is defined by an abrupt increase in kinetic energy. The increase in kinetic energy from an equilibrium state, under incremental loading, is shown to be equal to the difference between the external second-order work, involving the external loading parameters, and the internal second-order work, involving the constitutive properties of the material. When a stress limit state is reached, a certain stress component passes through a maximum value and then may decrease. Under such a condition, if a certain additional external loading is applied, the system fails, sharply increasing the strain rate. The internal stress is no longer able to balance the external stress, leading to a dynamic response of the specimen. As an illustration, the theoretical framework was applied to the well-known undrained triaxial test for loose soils. The influence of the loading control mode was clearly highlighted. It is shown that the plastic limit theory appears to be a particular case of this more general second-order work theory. When the plastic limit condition is met, the internal second-order work is nil. A class of incremental external loadings causes the kinetic energy to increase dramatically, leading to the sudden collapse of the specimen, as observed in laboratory.

Keywords: Failure in geomaterials; Undrained triaxial loading path; Second-order work; Kinetic energy; Plastic limit condition; Control parameter

1. Introduction

The notion of failure can be encountered in many fields, irrespective of the scale considered. This notion is essential in material sciences where the failure can be investigated on the specimen scale (the material point). It is also meaningful in civil engineering with regard to preventing or to predicting the occurrence of failure on a large scale.

If the definition of failure seems meaningful in some cases, at least from a phenomenological point of view, this is not always true, particularly, when considering heterogeneous materials. With regard to a granular assembly on a microscopic scale, failure might be related to the contact opening between initially contacting grains. However, the kinematic investigation of granular materials, along any given loading path, reveals that a large fraction of the contacts open without any visible failure pattern observed on the macroscopic scale. Thus, the usual view of failure as the breakage of a given material body into two pieces cannot be applied to complex, divided materials made up of an assembly of elementary particles or sub-systems described as approximately non-breakable.

For this reason, a mathematical definition of failure has emerged for solid materials. This definition was progressively built upon the plasticity theory, and developed early in the

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* Corresponding author.
E-mail address: francois.nicot@irstea.fr (François Nicot).
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20th century in the field of metallic materials. Failure means that the external stress applied cannot be increased, and that finite strains may develop and progress in a constant stress state.

For rate-independent materials, before failure occurrence along a given loading path, a unique strain state exists under a given external stress applied to the materials. This mathematical definition, applied to the case of a material point, leads to the following equations:

$$\begin{cases} \dot{\sigma}_i = 0 \\ N'_{ij} D_j = 0 \end{cases} \tag{1}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{D}$ is the strain rate tensor, and $\boldsymbol{N}'$ is the tangent stiffness operator. The constitutive behavior for this material point is defined by the incremental constitutive relation $\dot{\sigma}_i = N'_{ij} D_j$ between the strain rate and a suitable objective time derivative of the Cauchy stress expressed with two six-component vectors $\boldsymbol{D}$ and $\boldsymbol{\sigma}$ (Darve, 1990; Wan et al., 2011), which requires that

$$\det \boldsymbol{N}' = 0 \tag{2}$$

Eq. (2) is the failure criterion, and the equation $N'_{ij} D_j = 0$ corresponds to the associated failure rule. The failure rule gives the strain rate in different directions once failure has occurred. It should be emphasized that the magnitude of the strain rate remains unknown, and only the direction is determined, in accordance with the kernel of the tangent stiffness operator $\boldsymbol{N}'$.

Basically, the plastic surface splits the stress space into two parts: the inner part and outer part. The inner part (plastic domain) includes the stress states that can be reached by the materials. The stress states located on the plastic surface are therefore referred to as the stress limit states. The classical theory states that failure occurs in a given stress state which is located on the plastic surface. According to this theory, no failure mode is expected to occur in a mechanical state inside the plastic surface.

One famous counter-example is the liquefaction of loose sands along axisymmetric isochoric triaxial paths (Castro, 1969; Lade, 1992; Darve, 1990). During this test, the volume of the specimen is kept constant (isochoric conditions), and a constant axial displacement rate is imposed. This experiment shows that the curve giving the changes in the deviatoric stress q, defined as the difference between the axial stress σ_1 and the lateral stress σ_3, against the mean effective pressure p' passes through a maximum value, as shown in Fig. 1. If the test is strain-controlled by an imposed constant axial strain rate, the test can be pursued beyond the deviatoric stress peak until the collapse of the specimen: both the deviatoric stress q and the mean effective pressure p' decrease and tend toward zero. This is the well-known liquefaction phenomenon. Otherwise, when the deviatoric stress peak is reached, if an infinitesimal axial load is added, i.e., the strain control is replaced with a stress control, then a sudden failure occurs. Clearly, such experimental evidence evokes the notion of failure. According to the classical theory, the fact that the deviatoric stress peak (point P in Fig. 1) remains strictly inside the plastic domain means that no failure is supposed to occur at this point. However, the experiment shows that a proper loading control (namely, a stress control) can cause a sudden failure, leading to the collapse of the specimen.

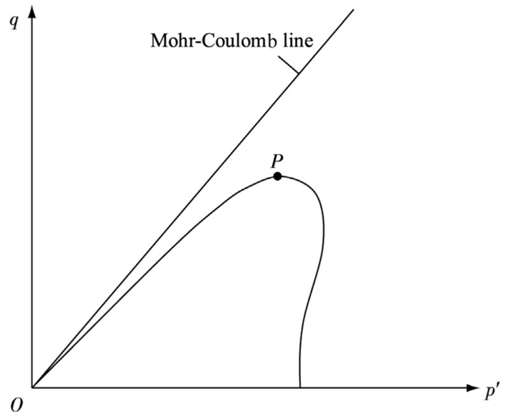

Fig. 1. Typical undrained triaxial behavior of loose sands.

As a consequence, the classic theory is not general enough, and a variety of failure modes may finally occur strictly within the plastic surface. Plastic failure, detected by Eq. (2), is a particular failure mode. Other failure modes may be encountered as well before the plastic limit is reached (such as the failure mode occurring at point P in Fig. 1). The detection of these failure modes requires a novel framework, which should lead to a different criterion from that given in Eq. (2).

Before proceeding with a description of the novel framework's construction, we have to propose a clear definition of failure based on phenomenological arguments. The weakness of the classical theory lies in the fact that it is not based on a physical definition of failure, but rather on a mathematical concept of the stress limit state. The approach presented in this paper allows the recovery of the notion of a limit state in a broader way, as a consequence of the theory, but not as a basic definition.

As far as non-viscous materials are concerned, we conceive that failure is related to a transition from a quasistatic regime toward a dynamic regime, giving rise to a sudden acceleration of the material points: the kinetic energy of the system evolves from a nil value to a strictly positive one. Thus, the failure is not a state, but a transition (bifurcation) from a quasistatic regime with a nil value of kinetic energy toward a dynamic regime with a non-zero value of kinetic energy. The failure occurs in a given mechanical state, which will be described as being potentially unstable: an increase in kinetic energy may take place under the loading conditions (Nicot et al., 2009, 2012).

In conclusion, the following definition around the notion of failure can be proposed: a material point in a given mechanical (stress–strain) state after a given loading history is described as being mechanically unstable as loading conditions lead to a bifurcation from a quasistatic regime toward a dynamic regime. This transition corresponds to a failure mode of the material.