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A procedure of strain-softening model for elasto-plastic analysis of a circular opening considering elasto-plastic coupling

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ABSTRACT

A simple numerical procedure of strain-softening model for the elasto-plastic analysis of a circular opening is presented by modifying the procedure proposed by Lee and Pietruszczak (2008). The proposed procedure has two advantages over that of Lee and Pietruszczak (2008). One is that the elasto-plastic coupling can be considered, the other is that the equilibrium equation and compatibility equation need not be expressed with respect to the normalized radius. Through MATLAB programming, the accuracy and validity of the proposed procedure are demonstrated through some examples. The influence of elastic modulus and Poisson's ratio evolution on the distribution of radial displacement and hoop stress, ground response curve (GRC) and plastic radius was investigated, and the results show that, with the decrease of elastic modulus evolutional ratio, the radial displacement, hoop stress and plastic radius all increase. With the increase of Poisson's ratio evolutional ratio, the radial displacement, hoop stress and plastic radius all decrease. As for influence extent, the evolution of elastic modulus has great influence on the radial displacement; and the evolution of Poisson's ratio has smaller influence on the radial displacement, both of them have very slight influence on the hoop stress near the excavation face but greater on the hoop stress near the interface of elastic and plastic region. And both of them have very slight influence on the plastic radius.

1. Introduction

Accurate acquisition of stress and displacement distribution around circular opening in isotropic rock mass has been one of the fundamental problems in underground engineering. The ground reaction curve of surrounding rock mass is usually required to be given to meet the support optimization design. In the past, to analyze the excavation problem of circular opening, elastic-perfectly plastic (Carranza-Torres et al., 2002), elastic perfectly brittle (Sharan, 2003; Jiang et al., 2007; Zhang et al., 2012a,b) and strain-softening models (Duncan, 1995; Guan, 2007; Carranza-Torres, 1998; Alonso et al., 2003; Park et al., 2008; Lee et al., 2008; Wang et al., 2010; Zhang et al., 2012a,b) all have been employed. In regard to application scope of the above three models, Hoek and Brown (1997) divide them into three cases according to geotechnical quality, and it is pointed out that elastic-perfectly plastic model is suitable for poor quality rock mass, elastic perfectly brittle is suitable for very high quality rock mass and strain-softening model is suitable for average quality rock mass. In fact, strain-softening model can accommodate elastic-perfectly plastic model (strain softening with a drop modulus equal to null) and elastic perfectly brittle model (strain softening with a drop modulus equal to infinity), i.e. elastic-perfectly plastic and elastic perfectly brittle models are extreme cases of the strain-softening model. When the strain-softening model is employed, the former studies have mainly considered the evolution of strength parameters such as cohesive, friction angle and so on, but the evolution of deformation parameters such as elastic modulus and Poisson's ratio, i.e. elasto-plastic coupling has seldom or only partially been considered as far. Actually, the elasto-plastic coupling exists in engineering and as illustrated in Fig. 1 (Hudson and Harrison, 2000). From Fig. 1 it can be seen that, due to the elasto-plastic coupling, the elastic modulus gradually decreases with the increase of strain in the post-peak region.

In this work, based on strain-softening model, a numerical procedure to calculate the distributions of displacement and stress around circular opening excavated in isotropic rock masses is developed by improving the procedure proposed by Lee and Pietruszczak (2008). The proposed procedure may consider the elasto-plastic coupling and is simpler than the procedure presented in Lee and Pietruszczak (2008). The accuracy and application of the proposed procedure are verified by some examples. The influence of elasto-plastic coupling on deformation and stress was discussed.

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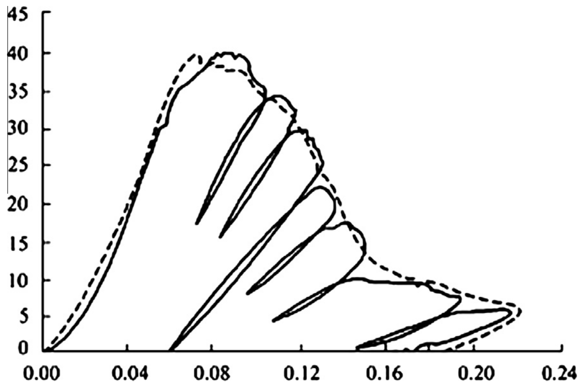

Fig. 1. Typical cyclic loading deformation curve of a rock sample.

2. Problem description

Fig. 2 shows a circular opening of radius R_0 excavated in an infinite isotropic rock mass, and an initial hydrostatic stress field σ_0 is imposed throughout the domain before the excavation. The internal supporting pressure p_0 is lower than a critical value p_{ic} to form a plastic region whose radius is denoted by R_p, and the p_{ic} can be calculated by use of the peak strength parameters and the initial hydrostatic stress σ_0 (Brady and Brown, 1993). When plastic region is formed, in some cases, the plastic region may be divided into softening and residual regions by an interface whose radius is denoted by R_s, and the radial stress σ_R, acting on the elastic–plastic interface, is equal to p_{ic}. Since strain-softening model can accommodate elastic-perfectly plastic model and elastic perfectly brittle model, here strain-softening model is employed.

2.1. Softening parameter, yield criterion and plastic potential function

In respect of studying the strain-softening behavior, the softening parameter, yield criterion and plastic potential function require to be determined firstly. Here, the most widely used plastic shear strain γ^p is selected as strain-softening parameter defined as

$$\gamma^p = \varepsilon_\theta^p - \varepsilon_r^p \tag{1}$$

where ε_θ^p and ε_r^p respectively denote the hoop and radial plastic strains.

The yield criterion is assumed by the following form

$$\sigma_\theta - \sigma_r = H(\sigma_r, \gamma^p) \tag{2}$$

where σ_θ and σ_r respectively denote the hoop and radial stresses. For Mohr–Coulomb and Hoek–Brown yield criterions, the strength parameters m, s, a, σ_c, c and ϕ are all gradually evolve with the increase of strain-softening parameter γ^p. So, for Mohr–Coulomb yielding criterion, $H(\sigma_r, \gamma_p)$ in Eq. (2) becomes

$$H(\sigma_r, \gamma^p) = \left(\frac{1+\sin\phi(\gamma^p)}{1-\sin\phi(\gamma^p)} - 1\right)\sigma_r + \frac{2c(\gamma^p)\cos\phi(\gamma^p)}{1-\sin\phi(\gamma^p)} \tag{3}$$

For Hoek–Brown yield criterion, $H(\sigma_r, \gamma_p)$ in Eq. (2) becomes

$$H(\sigma_r, \gamma^p) = \sigma_c(\gamma^p)\left(m(\gamma^p)\frac{\sigma_r}{\sigma_c(\gamma^p)} + s(\gamma^p)\right)^{a(\gamma^p)} \tag{4}$$

The Hoek–Brown yield criterion is employed in the following numerical examples in this study.

The widely used Mohr–Coulomb type of plastic potential function is assumed in this study which can be expressed as

$$G(\sigma_\theta, \sigma_r, \gamma^p) = \sigma_\theta - k(\gamma^p)\sigma_r \tag{5}$$

where $k(\gamma^p)$ is the coefficient of dilation defined as

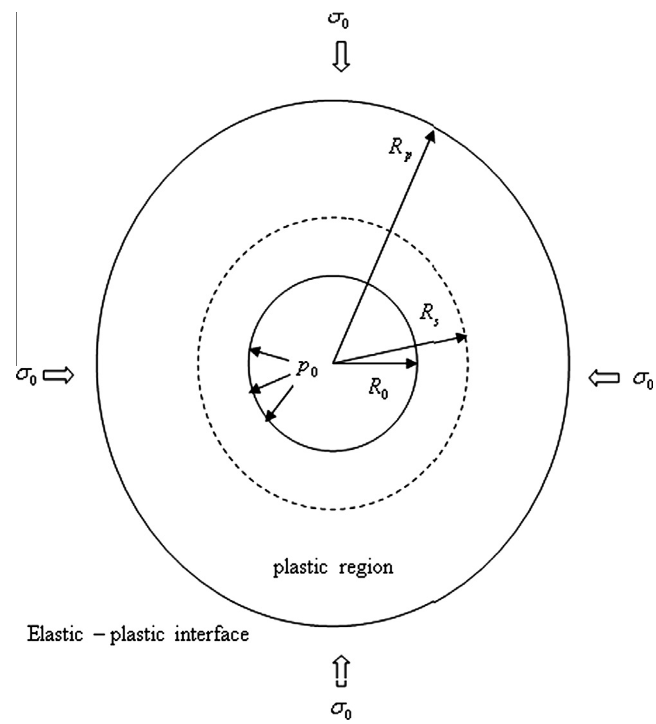

Fig. 2. Plastic region formed around circular opening.

$$k(\gamma^p) = \frac{1+\sin\varphi(\gamma^p)}{1-\sin\varphi(\gamma^p)} \tag{6}$$

where φ is dilation angle. So, according to the orthogonal plastic flow rule, the following relation between the radial and hoop plastic strain increments can be obtained:

$$d\varepsilon_r^p = -k(\gamma^p)d\varepsilon_\theta^p \tag{7}$$

2.2. Evolutional law

The relations of strength parameters and γ^p can be determined by laboratory experiment or field estimation. In our approach, it is assumed that strength parameters can be described by piecewise linear functions of γ^p as shown in express (8), both for the sake of simplicity and due to the fact that an analysis of tests in rock samples indicates a trend towards this linear decrease in the m s, a and σ_c parameters (Lee and Pietruszczak, 2008).

$$\eta(\gamma^p) = \begin{cases} \eta_p - (\eta_p - \eta_r)\frac{\gamma^p}{\gamma_r^p}, & 0 < \gamma^p < \gamma_r^p \\ \eta_r, & \gamma^p \geqslant \gamma_r^p \end{cases} \tag{8}$$

where η denotes one of m, s, a, σ_c and φ, γ_r^p is the critical plastic strain from which the residual region starts, the subscripts 'p' and 'r' denote the peak and residual values.

The effect of elasto-plastic coupling is not considered in the procedure proposed by Lee and Pietruszczak (2008). In order to consider the effect of elasto-plastic coupling, i.e. the effect of the evolution of elastic modulus and Poisson's ratio on deformation and stress in the plastic region, it is assumed that E and v can also be described by piecewise linear functions of γ^p shown as followers.

$$E(\gamma^p) = \begin{cases} E_p - (E_p - E_r)\frac{\gamma^p}{\gamma_r^p}, & 0 < \gamma^p < \gamma_r^p \\ E_r, & \gamma^p \geqslant \gamma_r^p \end{cases} \tag{9}$$