ORIGINAL ARTICLE

The effects of first- and second-order gable bends on forces and moments generated by triangular loops

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Introduction: Triangular loops are frequently used for space closure. Studies of this loop have focused on dimensional and in-plane (second-order) gable-bend influences on the generated forces and moments, but there have been no investigations into the effects of out-of-plane (first-order) gable bends. Both bends are generally needed to accomplish tooth translation. The primary purpose of this project was to ascertain whether first- and second-order bend effects were uncoupled. **Methods:** Ninety triangular loops were divided into 9 groups with combinations of 0° and 30° first- and second-order gable bends in the anterior and posterior positions. Forces (F_x, F_y, F_z) and moments (M_x, M_y, M_z) generated along 3 mutually perpendicular axes—x (mesiodistal), y (occlusogingival), and z (buccolingual)—were measured, and moment/force ratios (M_z/F_x, M_y/F_x) were calculated. Statistical comparisons were made between the 9 groups and between activation distances. The Sidak multiple-comparison adjustment method was used to control the overall confidence level at 95%. **Results:** It was shown that the magnitude of M_z/F_x increased significantly with second-order gable bends but did not change with first-order bends. The opposite was found for M_y/F_x. **Conclusions:** Thus, in triangular springs, first- and second-order gable bends produce the desired effects without interfering with each other. (Am J Orthod Dentofacial Orthop 2006;129:54-59)

Tooth displacement with minimal tipping (ie, translation) is often required for space closure. This technically challenging procedure is commonly achieved with closing loops that generate the appropriate mesiodistal closing force (F_x) and the necessary concomitant moment about the buccolingual axis (M_z) to counteract the tipping effect of F_x, (Fig 1, *A*). The ratio of the magnitudes of that moment and that force, the moment-to-force (M/F) ratio, is a critical spring-design parameter. Because these 2 load components (F_x and M_z) play the dominant roles in space closure, they have been extensively researched in the realms of orthodontic biomechanics and spring engineering. Basic statics calculations demonstrate that the M_z/F_x ratio must be equal to the occlusogingival distance between the tooth's center of resistance (CRes) and the bracket, about 10 mm for canine retraction. The questions raised in many research projects involve the abilities of various spring designs to generate such relatively large ratios, and what spring characteristics determine the ratio. Burstone and Koenig[1] showed that the M/F ratio increased with loop height and gingival side width and decreased with occlusal side width. Thus, the T and triangular loops are favorable designs for generating high M/F ratios. Not surprisingly, the 2 loops display similar behaviors.[2,3] The triangular loop, however, has the advantage of simpler fabrication.

For conceptual simplicity, a frequently overlooked aspect of tooth movement is that it is a 3-dimensional phenomenon. Teeth generally translate in 3 directions and rotate about those 3 directions. Corresponding to these 6 degrees of freedom are 3 force components and 3 moment components (Fig 1, *A*). We define the x, y, and z axes as the mesiodistal, occlusoapical, and buccolingual directions, respectively. Furthermore, mesial, occlusal, and buccal are the positive directions. As in previous studies,[2-4] the model is a mandibular right quadrant.

The most intuitive approach to discussing orthodontic tooth movement is to consider the equivalent force-moment system that acts at CRes. Relative to CRes, each of the 3 force components on the bracket generates moments about the other 2 axes. That is, F_x not only generates a moment (M'_z) about the z-axis, as

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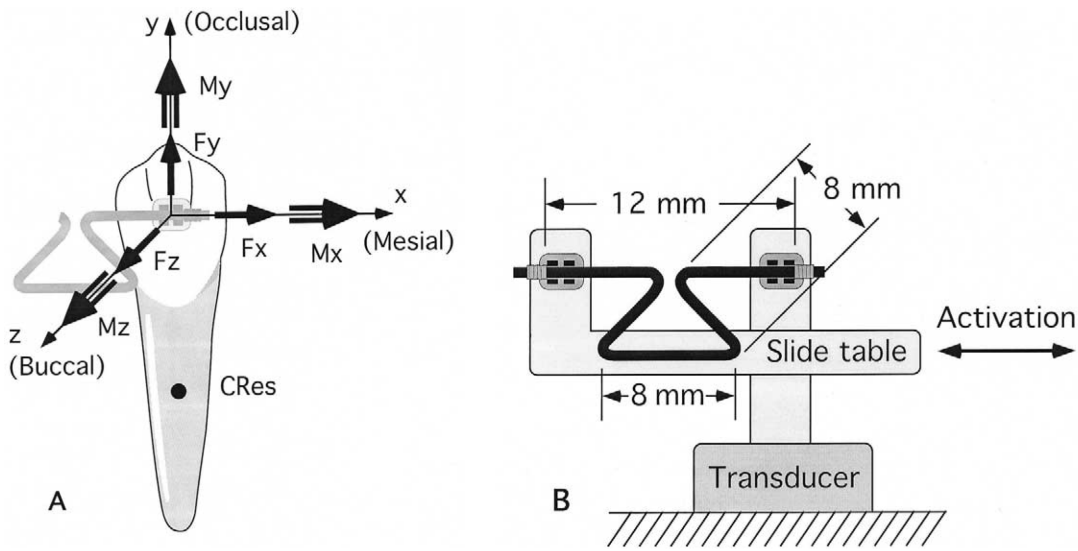

Fig 1. A, Spring-generated force and moment components that act on bracket; **B**, schematic of testing apparatus and spring.

discussed above, but also produces an M'_y component, a moment about the y-axis. F_x does not contribute to M'_x; F_y and F_z do. Also contributing to M'_z is F_y, and so on. With mathematical rigor, this can be expressed as $\mathbf{M}' = \mathbf{r} \times \mathbf{F}$, where $\boldsymbol{M}'$ is the moment vector relative to CRes, $\boldsymbol{r}$ is the position vector of the bracket relative to CRes, $\times$ is the vector (cross) product, and $\boldsymbol{F}$ is the force vector acting on the bracket:

$$\begin{aligned} \mathbf{M}' &= (\mathbf{r} \times \mathbf{F}) \\ &= (M'_x)\hat{x} + (M'_y)\hat{y} + (M'_z)\hat{z} \\ &= (r_yF_z - r_zF_y)\hat{x} + (r_zF_x - r_xF_z)\hat{y} \\ &\quad + (r_xF_y - r_yF_x)\hat{z} \end{aligned}$$

or

$$\begin{aligned} M'_x &= (r_yF_z - r_zF_y), \\ M'_y &= (r_zF_x - r_xF_z), \\ M'_z &= (r_xF_y - r_yF_x), \end{aligned}$$

where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are unit vectors. When pure translation is desired, $r_y > r_x$ and $r_y > r_z$, where r_y is the occlusogingival distance of the bracket from CRes, and r_x and r_z are the mesiodistal and buccolingual offsets of the bracket from CRes, respectively. Also, the closing force, F_x, is greater than F_y (the extrusive-intrusive force component) and F_z (the buccolingual force component). Thus, the expressions for the moment components can be simplified by eliminating second-order terms that are relatively small products of a small distance multiplied by a small force. This leaves the following nontrivial terms:

$$M'_x \approx r_yF_z,\ M'_y \approx r_zF_x,\ M'_z \approx -r_yF_x$$

M'_z dominates because it is the product of the largest r component (r_y) and the largest F component (F_x). It is for this reason that the M/F ratio that is typically addressed in the literature is r_y ($= M'_z/F_x$). This project also involves M'_y, the component that tends to rotate the tooth about its long axis. M'_y is produced by the closing force (F_x) as it acts on a bracket that is offset buccolingually from CRes by a distance r_z. Thus, for translation, the clinical challenge is that the spring must not only supply the closing force, F_x, but also moments (M_z and M_y) to counter those (M'_z and M'_y) that are produced relative to CRes primarily by the action of F_x. The spring-generated moment components at the bracket, M_z and M_y, must be equal in magnitude, but opposite in direction to M'_z and M'_y, respectively. This is generally achieved with the design of the spring and by the addition of gable bends. The associated design parameters are the M/F ratios r_z ($= M'_y/F_x$) and the previously discussed r_y ($= M'_z/F_x$). The main focus of this study is the interaction, if any, between the 2 counteracting M/F ratios produced by triangular loops. M_x is not addressed herein.

In the orthodontic literature, first-order bends are defined as producers of forces (F_z) and moments (M_y) that cause tooth movement in the buccolingual direction and rotation about the long axis of the tooth, respectively. Second-order bends produce forces (F_y) and moments (M_z) that cause movement in the occlusogingival direction and tipping of the root mesially or