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Reliability analysis of tunnels using a metamodeling technique based on augmented radial basis functions

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ABSTRACT

Metamodeling techniques have been developed and used for years in engineering reliability analysis involving expensive response simulations. In practical tunnel engineering problems where finite element (FE) simulations are required, the limited state/performance functions are in general implicit and nonlinear, and it is difficult to apply traditional gradient-based or sampling-based reliability methods, especially for large-scale problems. There is a need to develop accurate and efficient metamodels for practical tunnel engineering applications. In this paper, a metamodeling technique for reliability analysis of tunnels was studied based on augmented radial basis functions (RBFs). With a relatively small size of samples, the RBFs were used to create accurate approximate functions for different types of responses including linear and higher-order nonlinear functions. With the RBF-based metamodel constructed to express a limit state/performance function, Monte Carlo simulations (MCS) were applied to evaluate failure probability. The failure probability and reliability index obtained using the RBF-based metamodeling method were found to have good accuracy with a reasonable number of sample points. The reliability analyses of two existing tunnel examples showed that the augmented RBF metamodeling approach was efficient and effective for tunnel engineering problems.

1. Introduction

A reliability analysis is used to assess the failure probability and level of safety of an engineering component or system, such as tunnels or underground structures. The calculation of failure probability P_F is the computation of a multidimensional probability integral (Ang and Tang, 1975; Madsen et al., 1986; Kiureghian et al., 1987), as:

$$P_F \equiv P(g(\mathbf{x}) \leqslant 0) = \int_{g(\mathbf{x}) \leqslant 0} p_X(\mathbf{x}) d\mathbf{x} \tag{1}$$

where $\mathbf{x}$ is an m-dimensional vector of input random variables, $g(\mathbf{x})$ is the limit state or performance function such that failure is defined as $g(\mathbf{x}) \leqslant 0$, and $p_X(\mathbf{x})$ is the joint probability density function (PDF) of the random variable vector $\mathbf{x}$. In practical engineering applications, Eq. (1) is not straightforward to calculate since $p_X(\mathbf{x})$ is generally unknown. Moreover, due to the implicit nature, the limit of $g(\mathbf{x}) \leqslant 0$ is often difficult to formulate. Two types of commonly used reliability analysis methods are available: the *most probably point (MPP)-based methods* and the *sampling-based methods*. The MPP-based methods, such as the first-order or second-order reliability methods (FORM/SORM) are well known and widely used (Ang and Tang, 1975; Madsen et al., 1986; Kiureghian et al., 1987). Since the derivation and calculation of first-order sensitivities of system responses/simulation outputs is required, the integration of commercial FE programs with FORM/SORM is not easy, especially for nonlinear transient problems and coupled problems. The direct *sampling-based methods* such as Monte Carlo simulations (MCS) require the sampling of basic input random variables and calculating the limit state/performance function repeatedly (Rubinstein, 1981; Au and Beck, 2001). Since the sampling methods do not require sensitivity analyses of the limit state/performance function in terms of the random variables, a commercial FE program can be used as a black box. However the direct application of sampling methods for reliability analysis requires a considerable number of FE simulations; therefore the computational cost is very high when expensive FE simulations are required.

Reliability analysis of tunnels and underground structures has gained considerable attention in recent years using different reliability analysis methods (Hoek, 1998; Oreste, 2005; Li and Low, 2010; Mollon et al., 2009, 2011; Chen et al., 2010; Lv and Low,

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2011; Lv et al., 2011; Su et al., 2011; Zhang and Goh, 2012; Zhao et al., 2014). Hoek (1998) presented the reliability analysis of circular tunnels using MCS. Closed-from limit state/performance functions were employed. The circular tunnel example was also solved using FORM (Li and Low, 2010; Zhao et al., 2014). For most practical applications in tunnel engineering, a detailed FE simulation model is required in conjunction with reliability analysis methods. In order to reduce computational cost, the response surface methodology (RSM) is often used to create an explicit approximation of the implicit limit state/performance function with a simple polynomial (typically a quadratic) function (Mollon et al., 2009, 2011; Chen et al., 2010; Lv and Low, 2011; Lv et al., 2011). The RSM is the most commonly used metamodeling technique and has been used in a wide variety of engineering applications including reliability analysis (Faravelli, 1989; Rajashekhar and Ellingwood, 1993; Kim and Na, 1997; Gayton et al., 2003; Zheng and Das, 2000; Gavin and Yau, 2008; Kang et al., 2010). In the metamodeling approach, the expensive FE simulations are replaced by an approximate metamodel in which the response is a function of design variables or simulation inputs. The metamodel is explicit and very efficient to compute its values for a given set of input variables. In a reliability analysis, FORM/SORM can then be performed on the metamodel to calculate the reliability index (Lv and Low, 2011; Lv et al., 2011). Although an RSM model is simple and efficient, it generally cannot provide the required accuracy for highly nonlinear responses due to the use of a single, typically low-order polynomial to represent the entire input space. To improve the accuracy of global RSM models, various techniques were developed in the literature including vector projection sampling techniques (Kim and Na, 1997), resampling techniques (Gayton et al., 2003), and inclusion of higher order effects (Zheng and Das, 2000; Gavin and Yau, 2008). Local or successive RSMs were also proposed and applied in engineering reliability analysis, such as the moving least square technique (Kang et al., 2010).

Besides the conventional polynomial-based RSM, other types of metamodeling techniques have also been developed, including artificial neural networks (ANN) (Gomes and Awruch, 2004), Kriging (Jin et al., 2001; Zhang et al., 2011), high-dimensional model representation (HDMR) (Chowdhury et al., 2009), support vector machines (SVM) (Zhao et al., 2014; Tan et al., 2011), and radial basis functions (RBFs) (Bai et al., 2012; Krishnamurthy, 2003). One of the advantages of the RBF models is that there are no errors on sample points. Recent research results showed that the RBFs generated more accurate metamodels than the global RSM for highly nonlinear functions (Fang and Horstemeyer, 2006; Fang and Wang, 2008). There are different basis functions for creating RBF metamodels, whose accuracy largely depends on the selection of the basis functions. A thorough study of most available basis functions, including those commonly used and the compactly supported basis functions (Wu, 1995), was conducted by the authors (Fang and Horstemeyer, 2006; Fang and Wang, 2008). In the study, linear and high-order nonlinear mathematical functions as well as nonlinear responses from real-world engineering problems were considered. From these test examples, several generally accurate basis functions were identified. The results showed that the augmented RBF models, particularly those constructed with compactly supported functions $\phi(2,0)$, $\phi(3,0)$, and $\phi(3,1)$, created very accurate metamodels for all the test problems. These accurate RBFs were adopted for multiobjective design optimization of complex engineering structures (Fang et al., 2005).

For most engineering reliability analyses, there is a need to develop accurate and efficient metamodels in order to lower the computational cost without compromising the accuracy of results. Limited research has been conducted to evaluate the accuracy and efficiency of the augmented RBF models, including those with the compactly supported basis functions, for reliability analysis in tunnel engineering. This paper presents a study of the reliability analysis of tunnels using a metamodeling technique based on augmented RBFs. The proposed method applies the augmented RBF technique to approximate the implicit limit state/performance functions. Three augmented RBF models, the multiquadric function and two of Wu's compactly supported RBFs (Wu, 1995), were used in the reliability analysis of this work. The MCS method was adopted to calculate the failure probability using the RBF metamodels. The method was employed to predict the failure probability and reliability index of two tunnel engineering problems. Wherever possible, the results were compared with those obtained using the direct MCS method without metamodels and FORM to evaluate the accuracy and computational efficiency of the augmented RBF-based methods. It is shown that the augmented RBF metamodels generally provide accurate approximations of the original limit state/performance functions. Compared to the direct MCS method without using metamodels, the proposed approach of RBF-based reliability analysis provides an efficient and effective means to estimate failure probability and reliability index using a reasonable number of sample points.

2. Reliability analysis based on augmented RBFs

2.1. Augmented RBF metamodels

Consider a vector of m input variables $\mathbf{x} = [x_1, x_2, \ldots, x_m]$ and an output response function $g(\mathbf{x})$, which is implicit for most engineering applications but can be computed numerically for a given input vector $\mathbf{x}$. Before a metamodel function $\tilde{g}(\mathbf{x})$ can be constructed, the values of $g(\mathbf{x})$ need to be obtained at some sample points using a design of experiments (DOE) method. With the known values of $g(\mathbf{x})$ corresponding to a given set of input variables, the approximate function can be constructed using RBFs as

$$g(\mathbf{x}) \approx \tilde{g}(\mathbf{x}) = \sum_{i=1}^{n} \lambda_i \phi(\|\mathbf{x} - \mathbf{x}_i\|) \tag{2}$$

where n is the number of sample points, $\mathbf{x}_i$ is the vector of input variables at the ith sample point, $\|\mathbf{x} - \mathbf{x}_i\|$ is the Euclidean norm, ϕ is a basis function, and λ_i is the coefficient for the ith basis function. Replacing $\mathbf{x}$ and $\tilde{g}(\mathbf{x})$ in Eq. (2) with the n vectors of input variables and corresponding function values leads to n equations

$$\begin{aligned} \tilde{g}(\mathbf{x}_1) &= \sum_{i=1}^{n} \lambda_i \phi(\|\mathbf{x}_1 - \mathbf{x}_i\|) \\ \tilde{g}(\mathbf{x}_2) &= \sum_{i=1}^{n} \lambda_i \phi(\|\mathbf{x}_2 - \mathbf{x}_i\|) \\ &\ldots \\ \tilde{g}(\mathbf{x}_n) &= \sum_{i=1}^{n} \lambda_i \phi(\|\mathbf{x}_n - \mathbf{x}_i\|) \end{aligned} \tag{3}$$

The matrix form of Eq. (3) is given as

$$\mathbf{g} = \mathbf{A}\boldsymbol{\lambda} \tag{4}$$

where $\mathbf{g} = [\tilde{g}(\mathbf{x}_1) \quad \tilde{g}(\mathbf{x}_2) \quad \ldots \quad \tilde{g}(\mathbf{x}_n)]^T$, $A_{i,j} = \phi(\|\mathbf{x}_i - \mathbf{x}_j\|)$ ($i = 1, 2, \ldots, n, j = 1, 2, \ldots, n$), and $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_n]^T$. Coefficients $\boldsymbol{\lambda}$ can be calculated by solving Eq. (3) using any of the numerical methods for solving system of linear equations.

Although the RBF metamodels given by Eq. (2) generally provide good fit for high-order nonlinear responses, they may be less accurate for linear responses (Krishnamurthy, 2003). To ensure that an RBF metamodel can produce accurate approximations for both low- and high-order responses, an augmented RBF model was developed by adding a linear or quadratic polynomial to $\tilde{g}(\mathbf{x})$ as