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A model to assess the response of an arched roof of a lined tunnel

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ABSTRACT

A model to analyze the response of an arched roof of a tunnel lining under a surface static loading is presented. It enhances a previous model by the authors, which is based on a discrete-continuous concept and is suitable for depths of burial at which 'arching' can develop. The current enhanced model takes into consideration the curvature of an arched roof of a lined tunnel. The proposed 2DOF system's stiffness includes the influences of the soil side pressure as well as the arched geometry of the roof. For the case of zero curvature the analytical solution for the mid-roof deflection and average contact pressure that has been derived converges to the solution of a flat roof. The case of a relatively shallow buried structure has been calibrated and then verified against published experimental results.

A case study shows that there is a certain opening angle of the roof at which the contact pressure has a maximum value. This angle coincides with the angle at which there is also a maximum value of the roof stiffness. However, it is also shown that approximately at this opening angle, the internal forces are minimal. It is therefore concluded that the average contact pressure is not necessarily the most important criterion for a design of an optimal shape of the roof. Furthermore, the angle that yields maximum contact pressure should be preferred for an optimal roof design. It is further shown that as the roof slenderness increases this optimal angle decreases.

1. Introduction

An important parameter in the design of lined tunnels is the load acting on its roof. A common loading case is the self-weight of the covering or backfill soil above the structure and an external surface pressure. For deeply buried structures the weight of the soil upper layers may be regarded as a uniformly distributed load acting on the layers below them. The covering soil adjacent to the lining roof interacts with the structure in a way known for static loads as 'arching'. This phenomenon refers to a mechanism related to relative displacements in the soil media above the structure and far from it (e.g., Terzaghi, 1959; Newmark, 1964). Due to this phenomenon, the average contact pressure may be lower or higher than the undisturbed 'free-field' pressure. The first case is known as 'positive' or 'active' arching and the latter case is known as 'negative' or 'passive' arching.

A common way to evaluate the response of a buried structure is by numerical analysis. Finite element simulation of a specific problem is most frequently used (e.g., Brachman et al., 2000; Papanikolaou and Kappos, 2014; Mai et al., 2014,). These methods require proper representation of the soil and structure properties, as well as of the contact layer between them. They provide detailed analysis, but only for the specific problem that was analyzed. Thus, at the earlier stages of the design, when various alternatives need to be examined, such as structure geometry or element thicknesses, a theoretical model is preferred. Relatively simplistic models that are considered "classical" are known in the literature (e.g., Terzaghi, 1959; Newmark, 1964; Marston, 1930; Spangler, 1957). Simplistic models for structures with a rectangular cross-section have also been proposed (Weidlinger and Hinman, 1988; Higgins and Drake, 1995).

One of the frequently used shapes of tunnels is the horseshoe cross-section (Szechy, 1973). A model based on a discrete-continuous concept for the prediction of the average dynamic and static pressure acting on a flat roof tunnel has been developed by the authors (Dancygier and Karinski, 1999a,b; Karinski et al., 2003).

Unlike the models mentioned above, which refer only to buried structures with a flat roof, this paper proposes a model to assess the response of a lined tunnel with a non-flat, circular-arched roof under a surface static loading. It allows calculating the average contact pressure that acts on structures, such as horseshoe-roofed tunnel's lining. Although derivation of the model is relatively complicated, the resulted solution can be straightforwardly implemented. The paper starts with a description of the model,

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Nomenclature

A_a	cross sectional area of a unit width strip of the arch roof
b	width of the structure
D	depth of burial (from soil surface to the top of the arched roof)
E_s	Young's modulus of the structure roof
E_{soil}	Young's modulus of the soil-column above the structure
E^*	soil-column's equivalent modulus of elasticity
$E^{ff}_{soil}, E^{ff}_{b,soil}$	free field's moduli of elasticity above and below the level of the structure floor
E^{ff}, E^{ff}_b	free-field equivalent moduli of elasticity for a plane strain problem
h	thickness of the roof cross-section
H	total height of the structure's cross-section
I	moment of inertia of a unit width strip of the arch roof
k(x)	arching coefficient
k_0	constant of the arching coefficient function
k_s	side pressure coefficient
K, K_0	roof and floor stiffnesses
K_{msd}	measured roof stiffness
K_L	equivalent structure transformation factor
L	span of the arched roof and flat floor
M_b, m_b	bending moments caused by the contact pressure and by a vertical unit force acting at the top of the arch
M, M_0	2DOF masses and total masses of the structure roof and floor
N, n	axial forces caused by the contact pressure and by a vertical unit force acting at the top of the arch
P	roof perimeter
P_{ext}	external surface load (positive in compression)
P_{e0}	uniformly distributed load in the model (P_{e0} = -P_{ext})
P_v	total vertical contact force
q	uniformly distributed load acting on the roof (positive for pressure)
q_b	uniform pressure without taking into account free-field, $q_b = \tilde{q}_b + E^{ff} U^{ff}_x (D+H)$
$\tilde{q}_b$	average pressure that acts on the soil under the structure
R	radius of the roof arch
s	coordinate along the roof arch
S	horizontal projection of the area of the roof, which is also equal to the floor area
S_s	shear force
U(x)	vertical soil-column displacement above the structure
$U^{ff}(x)$	vertical free-field soil displacement
U_b	absolute mid-floor displacement
$U_x(x), U^{ff}_x(x)$	derivatives with respect to "x"
w, w_0	mid-roof and mid-floor deflections relative to their supports (walls)
W	modulus of a unit width strip of the arch roof
x	depth coordinate
y, y_0	roof and floor vertical deflection surfaces, relative to their supports (walls)
ν, ν_b	soil Poisson's ratios above and below the level of the structure floor.
α	coefficient in the settlement expression
β	factor of the arching coefficient exponent
γ_{soil}	soil weight density
μ	2DOF spring stiffness per unit area
ϕ_s	soil internal friction angle
θ	half of the arch opening angle
ρ	soil mass density
$\sigma(x)$	axial stress in the soil column
$\tau(x)$	friction traction on the perimeter of the soil column

followed by derivation of its equations and their solution. Finally, the solution is verified against experimental data and a case study is presented.

2. Description of the model

2.1. Basic discrete-continuous model

The work described herein is an extension of a model that was developed previously by the authors for flat roofs (Karinski et al., 2003). For the clarity of presentation, key parts of the original model are presented in the following text.

The model is suitable for depths of burial D, at which 'arching' (as explained above) can develop (at least about 15% of the roof span, e.g., see Dallriva and Hall, 1998). It comprises an equivalent structure subjected to a uniformly distributed equivalent load. The buried structure is represented here by an equivalent two-degree-of-freedom (2DOF) system interacting with an equivalent soil-column above it, where its Young's modulus is denoted here E_{soil}. At its bottom, the 2DOF system is supported by a semi-infinite elastic medium, Fig. 1a and b. It is assumed that there is a full (perfect) contact between the structure and surrounding soil. The soil at the sides of the structure and far from it is represented in the model by the 'free-field' stress and displacement expressions (as shown in Karinski et al., 2003). The shear soil resistance (τ) is represented in the model by a vertical friction traction that acts on the soil column perimeter and depends on the soil properties and on the relative displacement between the soil-column U(x) and that of the free field $U^{ff}(x)$ (where x is the depth, which is equal zero at the free surface). The coefficient of this relation k ('arching coefficient') may be either constant (Dancygier and Karinski, 1999a,b; Karinski et al., 2003) or some given function of the depth x (Chacha, 2014). The free field displacement for the case of a uniformly distributed load P_{e0} acting on the surface of an infinite half plane is given by Karinski et al. (2003):

$$U^{ff}(x) = U_0 + \begin{cases} \frac{P_{e0}}{E^{ff}}x - \frac{\rho g}{2E^{ff}}x^2; & 0 \leqslant x \leqslant D+H \\ \frac{P_{e0}}{E^{ff}}x - \frac{\rho g}{2E^{ff}_b}x^2 - \rho g(D+H)\left(\frac{1}{E^{ff}} - \frac{1}{E^{ff}_b}\right)x + \frac{\rho g}{2}(D+H)^2\left(\frac{1}{E^{ff}} - \frac{1}{E^{ff}_b}\right); & D+H \leqslant x \end{cases} \quad (1)$$

where H is the total height of the structure's cross-section, ρ is the soil mass density, g is the gravitational acceleration and U_0 is a constant, which is discussed in the following text. In Eq. (1) $E^{ff} = E^{ff}_{soil}\frac{(1-\nu)}{(1+\nu)(1-2\nu)}$, $E^{ff}_b = E^{ff}_{b,soil}\frac{(1-\nu_b)}{(1+\nu_b)(1-2\nu_b)}$ are the equivalent free-field moduli of elasticity of the above and below the level of the structure floor, for a plane strain problem, where $E^{ff}_{soil}, E^{ff}_{b,soil}$ are the corresponding "real" free field's moduli of elasticity, and ν and ν_b are the corresponding Poisson's ratios.

Note that Eq. (1) has been derived with the classical sign agreement that a tension force is positive. In the current problem external pressure, acting on the soil surface, P_{ext} is positive in compression (pressure), and therefore, $P_{e0} = -P_{ext}$ in Eq. (1) and in Fig. 1b.

It should also be noted that the proposed model is linear-elastic and is suitable for relatively small deflections and deformations that correspond to a 'service-state' condition (as opposed to an 'ultimate state' analysis).