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Imperfect bonding effect on dynamic response of a non-circular lined tunnel subjected to shear waves

Xue-Qian Fang [a,*], He-Xin Jin [a], Jin-Xi Liu [a], Ming-Juan Huang [b]

[a] Department of Engineering Mechanics, Shijiazhuang Tiedao University, Shijiazhuang 050043, China
[b] School of Foundation, Hebei Institute of Architecture and Civil Engineering, Zhangjiakou 075000, China

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ABSTRACT

Combining the wave function expansion method and conformal transformation method, the dynamic stress around a non-circular tunnel with imperfect interface subjected to anti-plane shear waves is derived. The non-circular tunnel is mapped into an annular region, and the analytic solutions of stress and displacement solutions are expanded in terms of wave functions. By introducing the spring-type interface model, the coefficients are determined by satisfying the imperfect bonding conditions around the concrete lining. The distribution of tangential stresses on the imperfect interface is graphically illustrated, and the interacting effect of imperfect interface and incident wavelength is discussed in detail. The imperfect interface is revealed as a key factor dominating the seismic responses of a tunnel.

1. Introduction

Underground tunnels are widely utilized around the world for various purposes, including subways, underground hydropower, material storage and military engineering. The tunnel liners are designed to withstand the static overburden and dynamic loading. The strength prediction is critical for the appropriate measurements, optimum design and safe operation of tunnels.

In recent decades, the number of large underground tunnels has been growing significantly. Proper estimation on the stress and displacement variation under many kinds of loading is important for the optimal design of such tunnels. To ensure the safety of tunnel liners, lots of methods including theory and experimental work have been proposed to calculate the stresses and displacements in the surrounding rock and the lining. The shapes include circular tunnels (Hefny, 1999), rectangular tunnels (Lei et al., 2001), semi-circular tunnels (Exadaktylos and Stavropoulou, 2002), inverted U-shaped and notched circular tunnels (Exadaktylos et al., 2003).

Although numerical methods such as finite element method (Ren et al., 2005; Wang et al., 2014) and finite difference method (Sobótka et al., 2013) have been applied to solve the response of tunnel structures, analytical techniques remain the convenient and efficient ways of providing the direct qualitative insights into the physical mechanism in the tunnel lining. For circular tunnels, the analytical solutions of displacements and stresses have been extensively studied (Fahimifar et al., 2010; Wang et al., 2012). In engineering application, the non-circular tunnel is very common. For inverted U-shaped and notched circular tunnels, Exadaktylos and Stavropoulou (2002) applied the complex potential formulation together with the conformal mapping representation to study the stresses and displacements around the tunnels with rounded corners. Kargara et al. (2014) presented a semi-analytical elastic plane strain solution for the stress field around a lined non-circular tunnel subjected to uniform ground load.

The imperfect interface around the tunnels plays an important role in controlling the dynamic response of tunnels. In recent years, the analytical, experimental and numerical methods have been used to analyze the imperfect interface effect on the response of tunnels. Combining the wave function expand method and Biot's dynamic theory of poroelasticity, Hasheminejad and Komeili (2009) studied the dynamic response of an arbitrarily thick elastic homogeneous hollow cylinder, which is imperfectly bonded to the surrounding fluid-saturated permeable formation. Based on the complex variable method, the analytical solution of stress fields of a lined non-circular tunnel with full-slip interface was presented, and the contact stresses along the rock-lining interface were analyzed (Lu et al., 2015). By using a mechanically adjustable tunnel model, Leung and Meguid (2011) designed an experimental setup to simulate the initial lining pressure that results from shield tunneling, and a local separation between the lining and the

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* Corresponding author.
E-mail address: stduxfang@yeah.net (X.-Q. Fang).

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surrounding soil was introduced. However, no works have dealt with the imperfect interface around the non-circular tunnels subjected to dynamic loadings.

The main objective of this paper is to present the analytical solutions of displacements and stresses resulting from the imperfect interfaces around the non-circular tunnels under anti-plane shear waves. Combining the wave function expansion method and conformal transformation method, the analytical solutions of anti-plane and stresses in the lining and surrounding rock are obtained. The imperfect interface around the tunnel is modeled by a spring-type medium. Through numerical examples, the effect of imperfect interface on the dynamic stress under different wave frequencies is examined.

2. Problem formulation and basis solutions

In this paper, a lined non-circular tunnel with imperfect interface is considered, as depicted in Fig. 1. The tunnel is embedded in the rock mass, and it is assumed that the surrounding rock mass and the lining are both linear isotropic and elastic. These two isotropic homogenous regions (S_1 and S_2) are bounded by contours L_1 and L_2. An anti-plane shear wave with frequency of ω propagates in the rock mass. Due to the geometrical character and the character of applied loading, this problem can be simplified into a plane strain problem. The shear modulus and density of rock mass are denoted by μ_R and ρ_R, and those of the lining are μ_L and ρ_L.

For this anti-plane problem, only the out-of-plane displacement field is considered, i.e.,

$$u_x = u_y = 0, \quad u_z = w(x, y, t), \tag{1}$$

where u_x, u_y and u_z denote the displacements in the x, y and z directions.

The governing equation in the isotropic homogenous medium is described as

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{c_{sh}^2}\frac{\partial^2 w}{\partial t^2}, \tag{2}$$

where c_{SH} with $c_{SH} = \sqrt{\mu/\rho}$ is the wave speed of anti-plane shear waves, and t is the time.

The constitutive relations of anti-plane shear displacement are expressed as

$$\tau_{xz} = \mu\frac{\partial w}{\partial x}, \quad \tau_{yz} = \mu\frac{\partial w}{\partial y}, \tag{3}$$

To express the wave field around the non-circular tunnel, the complex variable $z = x + \mathrm{i}y$ and its complex conjugate $\bar{z} = x - \mathrm{i}y$ should be introduced. Then, the following relations can be obtained

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}, \quad \frac{\partial}{\partial y} = \mathrm{i}\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}\right), \tag{4}$$

The steady solution is investigated, i.e., $w = We^{-\mathrm{i}\omega t}$. Then, the governing equation can be rewritten as

$$\frac{\partial^2 W}{\partial z \partial \bar{z}} + \frac{k^2}{4}W = 0. \tag{5}$$

where $k = \omega/c_{SH}$ is the wave number in the medium.

To obtain the analytical solution of this non-circular tunnel, the conformal mapping method of a complex function (shown in Fig. 2) is employed to solve the forward problem

$$z = g(\zeta) = R\left(\zeta + \sum_{m=0}^{N} c_m \zeta^{-m}\right), \quad \zeta = \xi + \mathrm{i}\eta = \bar{\rho}e^{\mathrm{i}\theta}, \tag{6}$$

where ξ and η denote the rectangular coordinate axes in the ζ plane, $\bar{\rho}$ and θ are the polar coordinate axes in the ζ plane. The mapping function transforms the boundaries L_1 and L_2 in the z-plane into two concentric circles S_1 and S_2 with R_0 and unit radii in the ζ plane. Since the tunnel is symmetrical about the y-axis, the coefficients c_m must be real numbers. All coefficients (R and c_m) in the mapping function can be determined by optimization methods when the shape of the tunnel cross-section and support thickness are known (Lu, 1996). The value of N is related to the shape of the tunnel, and the truncation at 4 is enough for computation.

From Eq. (3), the shear stress in the ζ plane can be expressed, in the cylindrical coordinate system, as

$$\tau_{rz} = \frac{\mu}{2}\left(\zeta\frac{\partial W}{\partial z} + \bar{\zeta}\frac{\partial W}{\partial \bar{z}}\right), \quad \tau_{\theta z} = \frac{\mathrm{i}\mu}{2}\left(\zeta\frac{\partial W}{\partial z} - \bar{\zeta}\frac{\partial W}{\partial \bar{z}}\right). \tag{7}$$

where $\bar{\zeta}$ denotes the conjugate of variable ζ.

To obtain the analytical expression in the rock mass and lining, Eq. (5) is transformed into the equation about variables ζ and $\bar{\zeta}$,

$$\frac{1}{g'(\zeta)\overline{g'(\zeta)}}\frac{\partial^2 W}{\partial \zeta \partial \bar{\zeta}} + \frac{k^2}{4}W = 0, \tag{8}$$

where X' the prime denotes the derivative of X, and $\overline{X}$ is the conjugate of X.

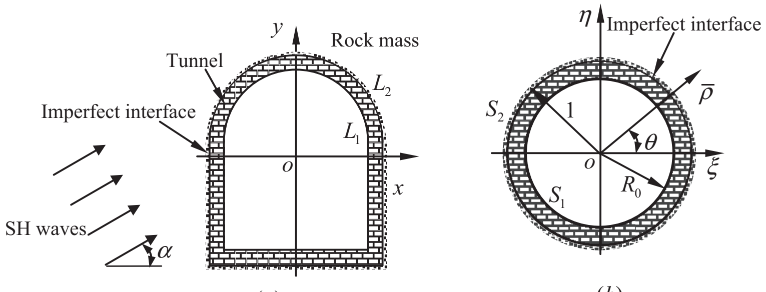
3. Wave fields and stresses in the rock mass and concrete lining

3.1. The incident wave and corresponding stress

The anti-plane shear waves with incident angle of α propagate in the rock mass. It is convenient to express the displacement in the cylindrical coordinate system, i.e.,

$$W^{(in)} = W_0 \sum_{n=-\infty}^{\infty} \mathrm{i}^n J_n(k_R|g(\zeta)|)e^{\mathrm{i}n(\theta-\alpha)}, \tag{9}$$

where W_0 is the amplitude of incident waves, k_R is the wave number in the rock mass, and $J_n(\cdot)$ is the nth Bessel function of the first kind.



(a) (b)

Fig. 1. A lined non-circular tunnel with imperfect interface under anti-plane shear waves. (a) Lined non-circular tunnel in the z plane; (b) Ring-shaped region in the ζ plane.