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A Graph-theoretic Pipe Network Method for water flow simulation in discrete fracture networks: GPNM

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ABSTRACT

In order to simulate water flow in discrete fracture networks, a Graph-theoretic Pipe Network Method (GPNM) is proposed. Firstly, identification of water flow pathways is considered and a tree cutting technique is adopted. Then each fracture in a discrete fracture network is treated as a weighted pipe with a starting node and an ending node in an oriented graph. A node law of flow rate and a pipe law of pressure in discrete fracture networks are derived based on the conservation of mass and energy, respectively. Boundaries and fractures are unified with the same form of a unified governing equation. Solutions of water pressures and flow rates in discrete fracture networks are obtained by solving a system of nonhomogeneous linear equations. Since no discretization is needed, GPNM is demonstrated with high efficiency. In addition, a few case studies are implemented and compared with those from analytical solutions or numerical analysis using the software, Universal Distinct Element Code (UDEC). It shows that the proposed Graph-theoretic Pipe Network Method (GPNM) is effective in analyzing water flow in discrete fracture networks. Moreover, GPNM is promising for more engineering applications, and can be used for large scales of water simulation problems with numerous fractures.

1. Introduction

Rock masses consist of various structural planes or discontinuities which form a discrete fracture network. The fracture network constructs potential fluid flow pathways in rock masses (Lapcevic et al., 1998). Fluid flow analysis is important in geotechnical and hydrogeological engineering. The fluid flow laws in discrete fracture networks (DFN) in rock masses are also influential in the selection of a site for a repository of nuclear waste, CO_2, petroleum and natural gas, for water supply reservoirs as well as hydraulic power stations, for the utilization of deep earth hot of geothermal reservoir systems and for water inflow assessment in underground engineering and mining (Caro Cuenca et al., 2013; Koh et al., 2011; Zhao et al., 2011).

Different conceptual and computational models have been established. Existing conceptual models differ in their representations of the heterogeneity of the fractured geological bodies, and they are classified into three broad classes: (1) single equivalent continuum models, (2) dual continuum models and (3) discrete fracture network models (de Dreuzy et al., 2013; Diodato, 1994; National Research Council (U.S.) 1996). Systematic descriptions and evaluations of these models can be found in open literature, such as flow phenomena in rocks: from continuum models to fractals, percolation, cellular automata, and simulated annealing (Sahimi, 1993), flow and transport in porous media and fractured rock: from classical methods to modern approaches (Sahimi, 2011), review of techniques, advances and outstanding issues in numerical modelling for rock mechanics and rock engineering (Jing, 2003; Jing and Hudson, 2002), characterizing flow and transport in fractured geological media (Berkowitz, 2002), trends, prospects and challenges in quantifying flow and transport through fractured rocks (Neuman, 2005). Single equivalent continuum models substantially simplify water flow problems on average, in which permeability is the sum effect of fracture and porous media. It is found that this approach does a fair job of conserving fluid mass where the scales of interest are sufficiently large. However, it is unacceptable in the presence of rapid flow transients, large fracture spacing, and low permeability rock matrix, spatial and temporal distribution of contaminant fluxes (Li et al., 2013; Long et al., 1982; Pruess et al., 1990a,b). Dual continuum models are based on two separate but overlapping porosities, a primary poros-

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ity in the rock matrix and a secondary porosity created by fractures. These models are good predictors while rock matrix accounts much of the porosity, but little of the permeability, which represents storage and fluid flow in geological bodies (Dershowitz and Miller, 1995; Haws et al., 2005; Huyakorn et al., 1983). Discrete fracture network (DFN) models are the most popular models for the simulations of water flow and transport in fractured rock masses (Andersson et al., 1984; Dershowitz and Einstein, 1987; Endo et al., 1984; Long et al., 1985; Robinson, 1984), and are useful for water flow analysis in fractured media where equivalent continuum parameters are hard to derivate and acquire (Bear et al., 1993; Zimmerman and Bodvarsson, 1995), or should turn to detailed site surveying and geophysical exploring (Niwas and Celik, 2012; Wishart et al., 2008). The advantage of DFN models is that they allow for explicit interpretation and representation of water flows in both fractures and rock matrix. Discrete fracture networks can be generated from site survey data by using the scan-line method, window mapping, borehole logging, image processing and statistical analysis to create probabilistic density functions (PDFs) of fracture parameters relating to the densities, orientations, spacing, locations and sizes (Billaux et al., 1989; Mauldon, 1998; Mauldon et al., 2001; Paillet, 1993; Tóth, 2010). However, data acquisition becomes onerous as the number of fractures and the complexity of the model increase, so that the computational burden also increases significantly (Diodato, 1994). Different computational models have been proposed based on the finite element method (FEM) (Dershowitz et al., 1993; Herbert, 1996; Wilcock, 1996), the boundary element method (BEM) (Elsworth, 1986a,b), the equivalent pipe model (EPM) (Cacas et al., 1990a,1990b) and other hybrid methods (Dershowitz and Fidelibus, 1999). Among these computational models, the Hybrid models are frequently used for hydraulic and mechanical problems in rock engineering, i.e. hybrids of BEM/FEM, BEM/DEM and DEM/FEM, in which the BEM is often used for simulating far-field rock masses as equivalent continuum, and the FEM or DEM for fractured media of the near-field with explicit representation and coupling of fractures and rocks. This harmonizes the geometry resolution with the numerical techniques available, thus providing an effective representation of both the far-field's effects and the near-field's, of the discontinuities' and the rocks' (Jing and Hudson, 2002).

The present Graph-theoretic Pipe Network Method (GPNM) has its own peculiar advantages. No discretization is needed in GPNM, which makes GPNM highly efficient while not sacrificing the accuracy, and so can be used for both the near and far-field simulations of water flow. Similar models to GPNM are the two dimensional fracture networks model (Long et al., 1982) and the pipe model (Cacas et al., 1990a,1990b). Conventional 2-D fracture networks models represent fractures as lines (pipes) in the 2-D domain (Long et al., 1982). The Cacas pipe model represents fracture networks as pipe networks, which assume that the flow occurs through bonds joining the center of each fracture disk to the center of the adjacent disks. The bonds are made up of two parts, one for each fracture, connecting the centers of the two adjacent fractures to the center of the intersection line (Cacas et al., 1990a, 1990b). However, GPNM graph-theoretically represents the physical model and the corresponding governing equations, while conventional 2-D fracture networks models and the Cacas pipe model solve all the pressures and velocities based on the FEM. In addition, boundary conditions as well as the porous medium are also unified as pipes in GPNM, which is completely different from conventional 2-D fracture networks. A unified governing equation of factures and boundaries is derived and integrated with topological graph theory, which has been used as the basis of mathematical models in many areas of engineering for its significant features, inherent, simple, and systematic manner. Thus, the method formulates the system equations directly from the description of the physical system (Jonathan and Thomas, 2001). A reference node is added into the graph-theoretic pipe network of the discrete fracture network, forming boundary pipes with the boundary nodes, making it possible to unify governing equations of both fracture pipes and boundary pipes in a connected, oriented and weighted network. Furthermore, corresponding basic laws and governing equations are studied. The graph-theoretic governing equation of water flow in both inner fractures and boundaries is derived and unified into a consistent format. Solutions of water pressures and flow rates in discrete fracture networks are obtained efficiently by solving a self-forming equation system. The graph-theoretical water flow analysis method is proposed for the calculation of steady-state pressure and flow rate distribution in discrete fracture networks. It is demonstrated that the Graph-theoretic Pipe Network Method (GPNM) has high efficiency and potential in engineering applications. It can also be extended to research areas such as assessment of other fluid or gas flow and transports in pipe networks and geological bodies.

The GPNM was originally termed the Topological Pipe Network Method (TPNM) in Xu's doctoral thesis (Xu, 2013). In reality, there is no topological operation, and the theory of an oriented graph of pipe network is used. GPNM covers points of both the physical mode, the topological pipe network and the computational model, the corresponding graph-theoretic laws of water flow. Therefore GPNM is the proper mathematical phrase.

2. Graph-theoretic Pipe Network Method

The Graph-theoretic Pipe Network Method is established based on the graph-theoretic characterization of water flow pathways. In the present study, each fracture is divided into several segments by the intersection nodes, and each segment is treated as a pipe with starting and ending nodes in a connected, directed and weighted graph (Jonathan and Thomas, 2001).

2.1. Tree cutting and identification of water flow pathways

Water flows in connected sub-pathways of discrete fracture networks. Fig. 1 schematically shows a 2-D fracture network. It

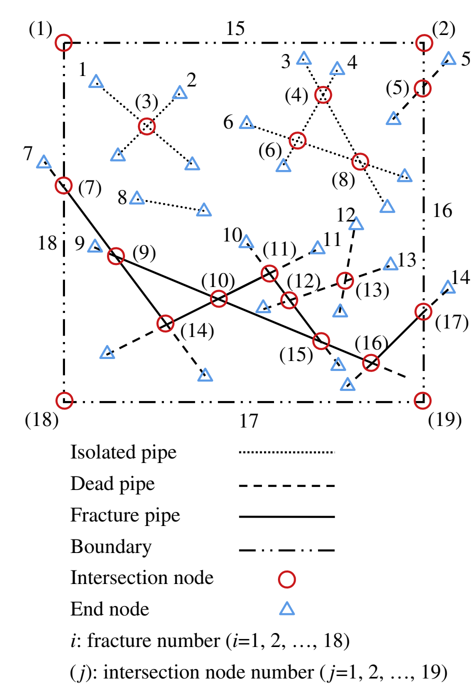

Fig. 1. Initial discrete fracture networks before tree cutting.