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Reliability analysis of tunnel using least square support vector machine

Hongbo Zhao [a,*], Zhongliang Ru [a], Xu Chang [a], Shunde Yin [b], Shaojun Li [c]

[a] School of Civil Engineering, Henan Polytechnic University, Jiaozuo 454003, People's Republic of China
[b] Department of Chemical and Petroleum Engineering, University of Wyoming, Laramie, WY 82071, USA
[c] State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan, Hubei 430071, People's Republic of China

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ABSTRACT

In the reliability analysis of tunnels, the limited state function is implicit and nonlinear, and is difficult to apply based on the traditional reliability method, especially for large-scale projects. Least squares support vector machines (LS-SVM) are capable of approximating the limited state function without the need for additional assumptions regarding the function form, in comparison to traditional polynomial response surfaces. In the present work, the LS-SVM method was adapted to obtain the limited state function. An LS-SVM-based response surface method (RSM), combined with the first-order reliability method (FORM), is proposed for use in tunnel reliability analysis and implementation of the method is described. The reliability index obtained from the proposed method applied to particular tunnel configurations under different conditions shows excellent agreement with Low and Tang's (2007) method and traditional RSM results, and indicates that the LS-SVM-based RSM is an efficient and effective approach for reliability analysis in tunnel engineering.

1. Introduction

Stability analysis is crucial in tunnel engineering. Numerical methods such as the finite element method and the discrete element method as useful tools have been used for stability analysis (Jing and Hudson, 2002). Limitations of these methods lie in lacking considerations of uncertainties in variables such as rock mass strength and the parameters of the support structure. To overcome above-mentioned limitations, the probabilistic approach has been developed for stability analysis of tunnels by combining with other techniques such as finite element (Hoek, 1998; Li and Low, 2010; Mollon et al., 2009, 2011; Oreste, 2005; Lv and Low, 2011; Su et al., 2011; Deng et al., 2005). The integration of analytical solutions and Monte Carlo simulation was applied to a reliability analysis of circular tunnels (Hoek, 1998). Oreste applied a probabilistic numerical approach to the design of primary tunnel supports (Oreste, 2005). Li and Low (2010) analyzed the stability of circular tunnels subjected to hydrostatic stress by combining a closed-form solution with a semi-probabilistic first-order reliability method (FORM). Mollon et al. (2009, 2011) studied the reliability of rock tunnel stability using a numerical model. Zhang and Goh (2012) estimated the stability of underground rock cavern using reliability method. From the practical point of view, the limit state surface is not known explicitly, whereas it is often known implicitly through a numerical procedure. A major difficulty in performing reliability analysis for realistic tunnel engineering problems is that the limited state function is not available as an analytical, closed-form function. To address this problem, the response surface method (RSM) has been employed to estimate the failure surface using an approximate closed-form expression (Mollon et al., 2009, 2011; Su et al., 2011; Lv and Low, 2011).

In traditional polynomial-based RSM, the number of samples required increase in tandem with the order of polynomial used. This can be time-consuming for practical engineering problems when a high-order polynomial is desired given a large number of input variables. Meanwhile, some studies have pointed out that it is most important to obtain support points for the response surface very close to or exactly at the limit state $g(x) = 0$ for reliability analysis (Kim and Na, 1997; Zheng and Das, 2000; Ouypornprasert et al., 1989). Artificial neural networks (ANN) and support vector machine (SVM) as the alternative approaches can be used to overcome the above problem (Bucher and Most, 2008; Zhao et al., 2009).

With the development of artificial intelligence, artificial neural network (ANN) model has been applied for reliability analysis in engineering. ANN-based response surface is able to avoid the problem of false design points arising from the use o polynomial response surface (Bauer and Pula, 2000). Deng et al. proposed an ANN-based second-order reliability method and an ANN-based Monte Carlo simulation method and applied in civil structure

* Corresponding author. Tel./fax: +86 3913987584.
E-mail address: bxhbzhao@hotmail.com (H. Zhao).

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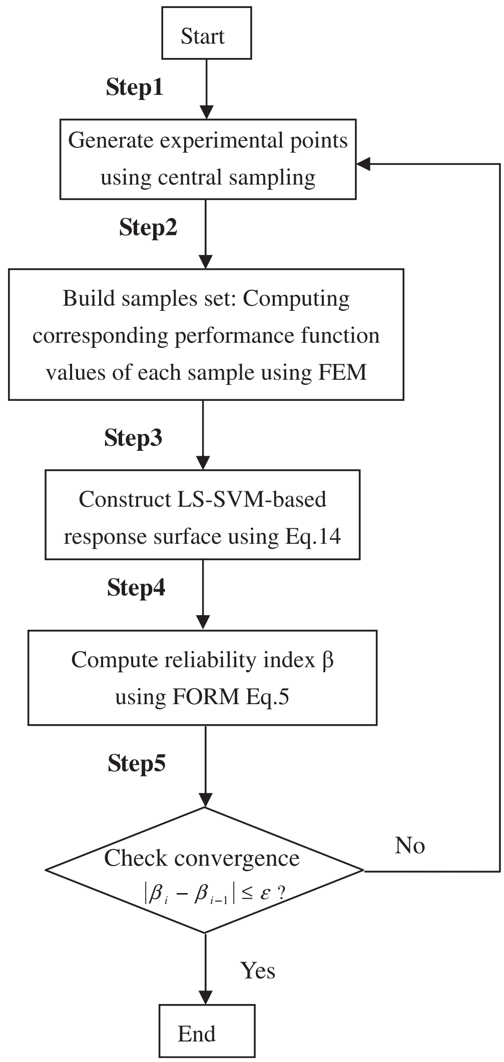

Fig. 1. Flowchart of LS-SVM-based response surface method.

engineering (Deng et al., 2005). Elhewy et al. proposed an ANN-based response surface method to analyze the reliability of structures (Elhewy et al., 2006). ANN-based response surface is adopted to approximate the limit state function and calculate the probability of failure through the first- and second-order reliability methods and a Monte Carlo simulation technique in order to reduce the number of stability analysis calculations (Cho, 2009). Other researchers have applied ANN to reliability analysis by combining it with Monte Carlo simulation, FORM, response surface method, etc. (Lopes et al., 2010; Cheng and Li, 2008; Cardoso, 2008). The performance comparison between the ANN-based RSM and the polynomial-based RSM showed that the ANN-based RSM is more efficient and accurate than the polynomial-based RSM (Gomes and Awruch, 2004; Elhewy et al., 2006). Response surface models in the form of ANN and SVM have the advantage of providing high-order approximations with smaller pools of samples compared to polynomial functions of comparable order (Gomes and Awruch, 2004; Deng et al., 2005; Zhao, 2008; Tan et al., 2011). However, ANN has some inherent drawbacks, such as its slow convergence, a less generalizing performance, arriving at a local minimum, and over-fitting problems. An alternative machine learning method, support vector machines (SVMs), is somewhat similar to ANN but appears to have more merit (Zhao and Yin, 2009).

Li et al. adopted SVM to structural reliability analysis through combining with FORM and MCSM. The results showed SVMs was more accurate and efficient in comparison to the traditional response surface method (Li et al., 2006). Zhao have used SVM-based FOSM and SVM-based Monte Carlo simulation to analyze the reliability of a slope and a tunnel respectively (Zhao, 2008; Zhao et al., 2009). In the previous study, Sequential minimum optimization (SMO)-based SVM algorithm was used to build the SVM model. SMO is an iteration algorithm which is time-consuming. To simplify the implementation of SVM, least squares support vector machines (LS-SVM) have been proposed as an alternative to SVM (Suykens and Vandewalle, 1999).

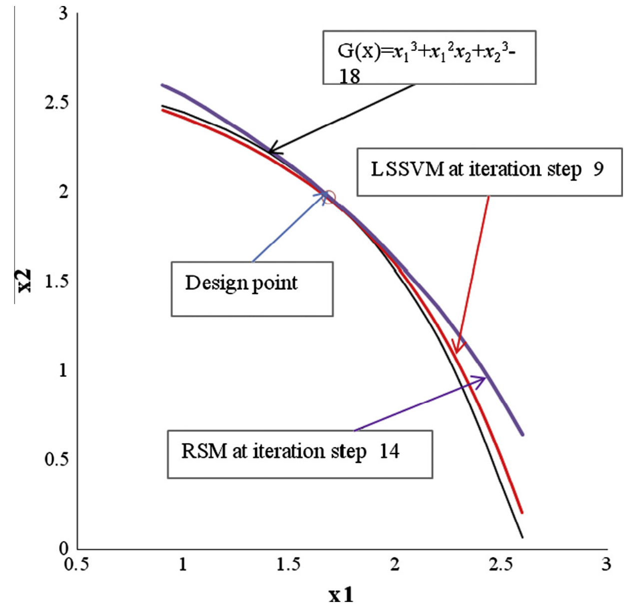

Fig. 2. The curve for verification example and its design point.

In this paper, a new approach that combines the merits of the response surface method and the least square support vector machine for reliability analysis is proposed. LS-SVM is capable of approximating the limited state function without more assumption of the function form. Then, the reliability index of tunnels was calculated by combining with the response surface method (RSM) and FORM. The paper is structured as follows: in Section 2, the FORM algorithm is reviewed; Section 3 introduces the basic theory of LS-SVM and RSM, and the LS-SVM-based reliability analysis method is proposed; and in Section 4, some applications of the proposed method to the reliability analysis of tunnels are presented. The Conclusions are given in Section 5.

2. FORM algorithms applying varying dimensionless numbers

Tunnel stability is affected by uncertain variables such as the elastic modulus of the rock mass, cohesion, friction angle and in situ stress. The reliability analysis methods quantify the combined uncertainties in a reliability index as a measure of uncertainty. The Hasofer–Lind index, β, is widely used in reliability analysis (Hasofer and Lind, 1974), for which the matrix formulation for a correlated normal is given by

$$\beta = \min_{X \in F} \sqrt{(X-\mu)^T C^{-1} (X-\mu)}, \tag{1}$$

where X is a vector representing the set of random variables x_i; μ is the vector of mean values; C is the covariance matrix; and F is the failure domain. Eq. (1) gives the minimum distance in units of directional standard deviations from the mean-value point of the random variables to the boundary of the limit state surface.