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Drained and undrained response of deep tunnels subjected to far-field shear loading

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ABSTRACT

An analytical solution for a rectangular opening in an infinite elastic medium subjected to far-field shear stresses has been derived for drained and undrained loading conditions. A number of numerical simulations has been conducted to determine the distortion of a rectangular structure in an infinite elastic medium under far-field shear stresses also for drained and undrained conditions and when there is full slip or no slip at the ground–structure interface. The results show that the shape of the opening has a minor influence on the structure's deformations and that full-slip conditions result in lower deformations. Undrained conditions tend to reduce distortions when the structure is more flexible than the ground, but tend to increase them for stiffer structures. A comparison between results obtained for a rectangular lined opening and for a circular lined opening are presented, and show that deformations of a rectangular structure with no-slip can be estimated from equations derived for a circular opening with an incompressible liner and also with no-slip. The effects of flexibility, slip condition at the interface, and drained or undrained loading are different for circular tunnels than for rectangular tunnels.

1. Introduction

The difference between above-ground and below-ground structures subjected to earthquake loading is that structures placed on the surface have a response determined by inertia forces and their displacements can be significantly different from those imposed by the ground. Design methods for aboveground structures typically involve the application of pseudo static forces, which approximate dynamic-induced forces. Underground structures however are constrained by the surrounding ground and thus it is unlikely that their displacements differ significantly from those of the ground. Hence, their analysis should be based on the displacements imposed by the ground rather than on inertia loading.

There are two basic approaches in present seismic design. One approach is to carry out dynamic, non-linear soil–structure interaction analysis using finite element or finite difference methods, where inertia forces are included. The input motions in these analyses are time histories emulating design response spectra. Input motions are applied to the boundaries of a soil island to represent propagating motion waves. In the second approach, the pseudo-static approach, inertia forces are neglected. The earthquake loading is simulated as a static far-field stress or strain applied to the ground where the structure is embedded. Hendron and Fernández (1983), Merritt et al. (1985) and Monsees and Merritt (1988) showed that the dynamic amplification of stress waves impinging on a tunnel is negligible when the rise time of the pulse is larger than about two times the transit time of the pulse across the opening; in other words, when the wave length of peak velocities is at least eight times larger than the width of the opening. In these cases the seismic load can be considered as a pseudo static load. This is an important conclusion, which has been used to derive simple analytical formulations for the seismic design of underground structures. It is important to note that a pseudo static analysis may be used for tunnels placed far from the seismic source, where frequencies of the ground motion are within the 0.1–10 Hz range.

Most of the numerical and analytical work has been done on underground structures assuming that during loading no excess pore pressures are generated, i.e. assuming a drained condition. While this is applicable to structures placed above the water table, the assumption may not be correct when the structure is placed below the groundwater table and is subjected to rapid loading. In this situation excess pore pressures may be generated in the surrounding ground, which do not dissipate during the loading event because of the short duration. This situation would correspond to undrained conditions.

This paper provides analytical and numerical results for deep underground structures with circular and rectangular cross sections subjected to a far-field shear stress or strain in undrained conditions. The results are compared with those obtained with the assumption of drained conditions. It is assumed that quasi-static loading approximates earthquake loading, with the understanding that the wavelength of peak velocities is large and warrants such assumption. It is also assumed that the ground and the liner, when present, remain within their elastic regimes.

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2. Elasticity equations for undrained conditions

For the derivations, the following assumptions apply: (1) plane strain conditions in a direction perpendicular to the cross section of the tunnel; (2) the ground is either dry or fully saturated, homogeneous, and isotropic; (3) deformations of ground and liner remain within their respective elastic regimes; (4) the tunnel is deep. Note that because of the assumption of deep tunnel, body forces, e.g. gravity, can be neglected.

The solution of any elasticity problem must satisfy the equilibrium equations, the strain compatibility equations, and the boundary conditions.

The equilibrium equations are:

$$\begin{aligned}&\frac{\partial\sigma' x}{\partial x}-\frac{\partial u}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0\\&\frac{\partial\sigma' y}{\partial y}-\frac{\partial u}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}=0\end{aligned}\tag{1}$$

which are written, as they should, in total stresses. σ'_x, σ'_y, and τ_{xy} are the cartesian effective normal and shear stresses, respectively; u is the pore pressure and x and y are the cartesian coordinates. It is assumed that the principle of effective stresses applies (e.g. Terzaghi et al., 1996), i.e.:

$$\begin{aligned}&\sigma'_x=\sigma_x+u\\&\sigma'_y=\sigma_y+u\end{aligned}\tag{2}$$

Note that Eq. (2) apply to ground with very compressible matrix and incompressible pore fluid, which is typical of soils and soft rocks. In the equation, tensile stresses are positive, compressive stresses are negative and pore pressures are positive in compression. This notation is followed through the paper.

The compatibility equation, in plane strain, takes the form:

$$\nabla^2\left[(\sigma'_x+\sigma'_y)-\frac{1}{1-\nu}u\right]=0\tag{3}$$

where ∇^2 is the Laplacian operator and ν is the Poisson's ratio. In total stresses Eq. (3) is written as:

$$\nabla^2\left[(\sigma_x+\sigma_y)+\frac{1-2\nu}{1-\nu}u\right]=0\tag{4}$$

or

$$\nabla^2(\nabla^2\phi)=-\frac{1-2\nu}{1-\nu}\nabla^2 u\tag{5}$$

where ϕ is the Airy stress function and is defined in cartesian coordinates as (e.g. Timoshenko and Goodier, 1970)

$$\begin{aligned}&\sigma_x=\sigma'_x-u=\frac{\partial^2\phi}{\partial y^2}\\&\sigma_y=\sigma'_y-u=\frac{\partial^2\phi}{\partial x^2}\\&\tau=-\frac{\partial^2\phi}{\partial x\partial y}\end{aligned}\tag{6}$$

Pore pressures are governed by the following field equation:

$$\frac{K}{\gamma_w}\nabla^2 u=-\frac{\partial\zeta}{\partial t}\tag{7}$$

where K is the coefficient of permeability, γ_w is the unit weight of water, and ζ is the change of fluid volume per unit volume of the ground.

In general, mechanical-flow problems are coupled so Eqs. (5) and (7) must be solved simultaneously. There are few cases however where the equations can be decoupled. The first case is for dry ground, where pore pressures are zero. For this problem Eq. (7) has the trivial solution $u = 0$ and Eq. (5) recovers the classical form (Timoshenko and Goodier, 1970). The second case is for undrained, i.e. short-term, conditions. This situation is imposed with $\zeta = 0$; that is, there is no change of fluid in the ground. This occurs in materials with low permeability, where loading induces excess pore pressures at a rate much faster than the capability of the ground to dissipate them. The third and final case is for long-term conditions where all excess pore pressures have dissipated and variables do no change with time. In this case $\delta/\delta t = 0$. All three cases result in $\nabla^2 u = 0$, and so they must obey the following equation:

$$\nabla^2(\nabla^2\phi)=0\tag{8}$$

This is an important finding because it indicates that existing solutions that satisfy (8) may also satisfy undrained conditions, i.e. short-term analysis, provided that boundary conditions are satisfied. The constraint $\zeta = 0$ or no change in volume, in elasticity and given that the principle of effective stresses applies, requires:

$$\sigma'_x+\sigma'_y=0\tag{9}$$

which is then used to find pore pressures and effective stresses once the total stresses are known, e.g. from (8), as follows:

$$\begin{aligned}&\sigma'_x=-\sigma'_y=\frac{\sigma_x-\sigma_y}{2}\\&u=-\frac{\sigma_x+\sigma_y}{2}\end{aligned}\tag{10}$$

The Airy stress function has been successfully used to solve problems on deep tunnels with circular cross section. For rectangular cross sections, complex variable theory and conformal mapping techniques have been preferred (Mindlin, 1940, 1948; Muskhelishvili, 1954; Sokolnikoff, 1956; Theocaris and Petrou, 1989; Theocaris, 1991; Motok, 1997; Gerçek, 1991, 1997; Exadaktylos and Stavropoulou, 2002; Exadaktylos et al., 2003; Huo et al., 2006; Li and Wang, 2008). The fundamental theories of complex variable and conformal mapping have been extensively described by Muskhelishvili (1954), and later on by Savin (1961) and Timoshenko and Goodier (1970). For circular tunnels the method has been very effectively used by Verruijt (1997, 1998); Strack and Verruijt (2002).

According to complex function theory, any biharmonic function, for example the Airy stress function ϕ in Eq. (5), can be expressed as:

$$\phi=\mathrm{Re}[\bar{z}\varphi(z)+\chi(z)]\tag{11}$$

where z is a complex variable ($z = x + iy$; with $i=\sqrt{-1}$) and $\bar{z}$ is the complex conjugate of z ($\bar{z}=x-iy$);$\varphi(z)$ and $\psi(z)=\chi'=\frac{d\chi(z)}{dz}$ are two analytic complex functions, also known as "complex potential functions". The stress components, for example in a cartesian coordinate system, can be expressed in terms of the complex potentials as:

$$\begin{aligned}&\sigma_x+\sigma_y=2[\varphi'(z)+\overline{\varphi'(z)}]\\&\sigma_y-\sigma_x+2i\tau=2[\bar{z}\varphi''(z)+\psi'(z)]\end{aligned}\tag{12}$$

The displacements in plane strain are:

$$2G(U_x+iU_y)=(3-4\nu)\varphi(z)-z\overline{\varphi'(z)}-\overline{\psi(z)}\tag{13}$$

where U_x and U_y are horizontal and vertical displacements, respectively; G is the shear modulus of the medium, $G = E/[2(1 + \nu)]$ with E the Young's modulus and ν the Poisson's ratio.

For undrained conditions, given Eqs. (8)–(10), stresses and displacements are obtained solving the following two sets of equations: