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Effects of heterogeneity distribution on hillslope stability during rainfalls

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Abstract

The objective of this study was to investigate the spatial relationship between the most likely distribution of saturated hydraulic conductivity (K_s) and the observed pressure head (P) distribution within a hillslope. The cross-correlation analysis method was used to investigate the effects of the variance of $\ln K_s$, spatial structure anisotropy of $\ln K_s$, and vertical infiltration flux (q) on P at some selected locations within the hillslope. The cross-correlation analysis shows that, in the unsaturated region with a uniform flux boundary, the dominant correlation between P and K_s is negative and mainly occurs around the observation location of P. A relatively high P value is located in a relatively low K_s zone, while a relatively low P value is located in a relatively high K_s zone. Generally speaking, P is positively correlated with q/K_s at the same location in the unsaturated region. In the saturated region, the spatial distribution of K_s can significantly affect the position and shape of the phreatic surface. We therefore conclude that heterogeneity can cause some parts of the hillslope to be sensitive to external hydraulic stimuli (e.g., rainfall and reservoir level change), and other parts of the hillslope to be insensitive. This is crucial to explaining why slopes with similar geometries would show different responses to the same hydraulic stimuli, which is significant to hillslope stability analysis.

Keywords: Cross-correlation analysis; Heterogeneity; Hillslope stability; Saturated hydraulic conductivity; Stochastic conceptualization; Pore-water pressure

1. Introduction

Spatial variability of hydraulic properties of geologic media (e.g., the presence of macropores, fractures, folds, fissures, layers, preferential flow paths, and clay lenses) is the rule rather than the exception. Even seemingly uniform field sites show a high degree of spatial variation in the saturated hydraulic conductivity value (Nielsen et al., 1973). Such variability controls hillslope soil strength distribution, groundwater flow, pore-water pressure distribution, and seepage force, and, in turn, plays a salient role in hillslope stability along with other factors such as spatial and temporal variability of precipitation. As such, this information allows determination of stress distribution within a hillslope, and appropriate actions can be taken to mitigate possible failure of the hillslope. While the importance of the spatial variability or heterogeneity is well known, it is practically impossible to describe these heterogeneities in detail. In order to overcome this difficulty, probabilistic methods, or methods based on random or stochastic field theory, have been employed in hydrogeology and geotechnical engineering (Yeh, 1992; Gelhar, 1993; Griffiths and Fenton, 1993, 2004; Gui et al., 2000; Srivastava et al., 2010; Cho, 2012; Zhu et al., 2013; Yeh et al., 2015). For example, the random finite element method (RFEM), proposed by Griffiths and Fenton (1993) and Fenton and Griffiths (1993), has been widely used to consider the spatial fluctuations of a parameter

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in hillslope stability analysis (Gui et al., 2000; Cho, 2012, 2014; Zhu et al., 2013).

Generally, for a given hillslope, one may collect soil samples at some locations to determine spatial statistics of hydraulic parameters; these spatial statistics are then used to infer the spatial variability of hydraulic parameters across and throughout the hillslope. From the given spatial statistics of the parameters, one can generate many possible spatial distributions of the parameters of the hillslope. Of these distributions, some are favorable to slope stability, and some are not for given rainfall events. In practice, it is rather costly and difficult to characterize the hydraulic heterogeneity by taking soil samples; on the other hand, it is relatively inexpensive and typical to observe hydraulic responses/states, such as the pore-water pressure at hillslope toes or the position of phreatic surface at some locations. As a matter of fact, pore-water pressure distributions are one of the major stresses that control the hillslope stability. This simple reality thus compels us to ask, what can be inferred about the likely spatial distribution of saturated hydraulic conductivity within a hillslope from some observed hydraulic responses or pore-water pressures at some locations within the hillslope?

Zhu et al. (2013) carried out a probabilistic infiltration analysis considering a spatially varying permeability function within a two-dimensional (2D) hillslope. With one set of given statistical parameters, they showed the spatial distributions of saturated hydraulic conductivity that can lead to the highest and lowest groundwater tables in the hillslope. However, they did not focus on the effects of the spatial distribution of saturated hydraulic conductivity in various hydraulic scenarios, nor did they investigate the relationship between the likely distribution of saturated hydraulic conductivity and observed pore-water pressures at some given locations within a hillslope.

Cross-correlation analysis can serve as a quantitative tool for answering the question posed above. Cross-correlation analysis has been widely used by hydrogeologists over the past decade. For example, Yeh et al. (2014) used a simple example to elucidate the cross-correlation relationship between the observed heads in a saturated aquifer and the hydraulic conductivity in a one-dimensional aquifer. Mao et al. (2011) conducted cross-correlation analysis to investigate how the spatial variability of hydraulic parameters, such as the saturated hydraulic conductivity, specific storage, and saturated moisture content, at different locations in unconfined aquifers affect the head at a given location in the unsaturated and saturated regions. Wu et al. (2005) and Sun et al. (2013) used cross-correlation analysis to study the spatial and temporal evolution of cross-correlations between soil properties (transmissivity and storage coefficients) and the head responses at an observation well during a pumping test, in both homogeneous and heterogeneous aquifers.

At present, few have attempted to study the ways in which the saturated hydraulic conductivity at different locations within a hillslope influences the pore-water pressure at some crucial locations within the hillslope. More importantly, few studies have quantified the relationship between the most likely distribution of saturated hydraulic conductivity and the observed pressure head distribution within a hillslope subject to rainfalls or other external events. This relationship may allow us to better control the pore-water pressure distribution within a hillslope.

The objectives of this study were therefore (1) to examine how the pressure head at a crucial location (e.g., the slope toe) is affected by the saturated hydraulic conductivity at different locations within a hypothetical hillslope; (2) to investigate the relationship between the most likely distribution of saturated hydraulic conductivity and the observed pressure head distribution within a hillslope; and (3) to elucidate the likely distribution of saturated hydraulic conductivity and the flow field distribution within a heterogeneous hillslope subject to some given hydraulic events, which are critical to the study of hillslope stability.

2. Methodology

2.1. Governing equation

While transient flow is more realistic, for the sake of simplicity and a first cut analysis, in this paper we focus on steady-state flow in a heterogeneous hillslope, which is pertinent to long-term status analyses. Here, we assume that the flow in a heterogeneous 2D vertical hillslope cross-section can be described by the following equation:

$$\frac{\partial}{\partial x}\left[K(P)\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial z}\left[K(P)\frac{\partial (P+z)}{\partial z}\right]=0 \tag{1}$$

and is subject to the following boundary conditions:

$$\begin{cases} P(x,z)|_{\Gamma_{\mathrm{D}}}=P_{\mathrm{D}} \\ K(P)\dfrac{\partial P}{\partial x}n_x+K(P)\left(\dfrac{\partial P}{\partial z}+1\right)n_z\Big|_{\Gamma_{\mathrm{N}}}=q_{\mathrm{N}} \end{cases} \tag{2}$$

where P is the pressure head; z is the elevation head; $K(P)$ is the hydraulic conductivity-pressure constitutive function; P_{D} is the prescribed pressure head at the Dirichlet boundary Γ_{D}; q_{N} is the specific flux at the Neumann boundary Γ_{N}; and n_x and n_z are the components of the unit vector $\boldsymbol{n}$ in the x and z directions, respectively, and $\boldsymbol{n}$ is normal to the boundary Γ_{N}.

2.2. Moisture retention and hydraulic conductivity functions

In order to simulate flow in a hillslope using Eqs. (1) and (2), the moisture retention and hydraulic conductivity curves developed by van Genuchten (1980) and Mualem (1976), respectively, also known as the MVG model, are adopted here, and can be expressed as