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A comparative study for empirical equations in estimating deformation modulus of rock masses

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ABSTRACT

The deformation modulus of rock masses (E_m) is one of the significant parameters required to build numerical models for many rock engineering projects, such as open pit mining and tunnel excavations. In the past decades, a great number of empirical equations were proposed for the prediction of the rock mass deformation modulus. Existing empirical equations were in general proposed using statistical technique and the reliability of the prediction relies on the quantity and quality of the data used. In this paper, existing empirical equations using both the Rock Mass Rating (RMR) and the Geological Strength Index (GSI) are compared and their prediction performances are assessed using published high quality *in situ* data. Simplified empirical equations are proposed by adopting Gaussian function to fit the *in situ* data. The proposed equations take the RMR and the deformation modulus of intact rock (E_i) as input parameters. It has been demonstrated that the proposed equations fit well to the *in situ* data compared with the existing equations.

1. Introduction

The deformation modulus (E_m) is the most representative parameter of the mechanical behavior of rock masses. It is widely used in numerical modeling, such as finite element modeling, of rock engineering projects where the analysis of displacement and stress distribution are required to characterize the rock mass behavior.

Commonly used approaches to estimate E_m includes: laboratory tests, *in situ* loading tests and prediction by empirical equations. However, laboratory tests on limited size rock samples containing discontinuities cannot measure reliably values of E_m due to the limitation of size of the testing equipment (Palmström, 1996). *In situ* tests can provide direct information on the deformability of rock masses, however, as Bieniawski (1973) noted, it is difficult to rely on one *in situ* test alone as different results may be obtained even in a fairly uniform and good quality rock mass condition. Therefore, in order to obtain reliable results multi-tests are necessary which are expensive and time consuming.

Due mainly to the above mentioned difficulties encountered in laboratory and *in situ* testing, the estimation of E_m values using empirical equations becomes a very attractive and commonly accepted approach among rock engineers.

In the past decades, a great number of empirical equations were proposed for the estimation of the isotropic rock mass deformation modulus using various rock mass classification systems, such as the Rock Mass Rating (RMR), the Geological Strength Index (GSI) (see Table 1), the Tunneling Quality Index (Q) (Barton, 1987, 1996, 2002) and the Rock Mass Index (RMI) (Palmström, 1996; Palmström and Singh, 2001). Other authors proposed equations on the basis of parameters which define the quality of the rock masses, such as the Rock Mass Quality Designation (RQD) (Zhang and Einstein, 2004) and the Weathering Degree (WD) (Gokceoglu et al., 2003; Kayabasi et al., 2003).

Existing empirical equations were in general derived using statistical methods, such as the regression analysis, and the reliability of estimation of these equations depends on the quantity and quality of data used in the statistical analysis. As a consequence, large discrepancies in the predicted values using different empirical relations can be experienced which reduce the confidence in the predicted values. For example, for a rock mass with the following properties: GSI = 70, the disturbance factor, $D = 0$ and the intact rock deformation modulus, $E_i = 50$ GPa, the values of E_m calculated from the empirical equations proposed by Carvalho (2004), Sonmez et al. (2004) and Hoek and Diederichs (2006) (see Group 4 in Table 1) are 21.7 GPa, 25.6 GPa and 36.6 GPa, respectively. Clearly the reliability of the prediction of these empirical equations needs to be assessed.

In this research, existing empirical equations using the RMR and the GSI classification systems are evaluated. The prediction performance of these equations is tested by using high quality well publicized *in situ* data from Bieniawski (1978), Serafim and Pereira (1983) and Stephens and Banks (1989). These data are from high

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Table 1
Empirical equations using RMR and GSI for predicting E_m.

Input parameters	Empirical equations	
Group 1 RMR	Bieniawski (1978)	$E_m = 2\text{RMR} - 100, \text{RMR} > 50$
	Serafim and Pereira (1983)	$E_m = 10^{(\text{RMR-10})/40}$
	Mehrotra (1992)	$E_m = 10^{(\text{RMR-20})/38}$
	Read et al. (1999)	$E_m = 0.1(\text{RMR}/10)^3$
Group 2 RMR and E_i	Nicholson and Bieniawski (1990)	$E_m = 0.01E_i(0.0028RMR^2 + 0.9e^{\frac{RMR}{22.83}})$
	Mitri et al. (1994)	$E_m = E_i[0.5(1 - (\cos(\pi\text{RMR}/100)))]$
	Sonmez et al. (2006)	$E_m = E_i 10^{((\text{RMR-100})(100\text{-RMR}))/(4000\exp(-\text{RMR}/100))}$
Group 3 GSI and D	Hoek et al. (2002)	$E_m = (1 - 0.5D)10^{\left(\frac{\text{GSI}-10}{40}\right)}, \sigma_{\text{ci}} > 100\ \text{MPa}$
	Hoek and Diederichs (2006)	$E_m(\text{MPa}) = 10^5\left(\frac{1-0.5D}{1+e^{((75+25D-\text{GSI})/11)}}\right)$
Group 4 GSI, D and E_i	Carvalho (2004)	$E_m = E_i(s)^{0.25}, \quad s = \exp\left(\frac{\text{GSI}-100}{9-3D}\right)$
	Sonmez et al. (2004)	$E_m = E_i(s^a)^{0.4}, \quad s = \exp\left(\frac{\text{GSI}-100}{9-3D}\right) \quad a = 0.5 + \frac{1}{6}(e^{-\text{GSI}/15} - e^{-20/3})$
	Hoek and Diederichs (2006)	$E_m = E_i\left(0.02 + \frac{1-0.5D}{1+e^{((60+15D-\text{GSI})/11)}}\right)$
Group 5 GSI, D and σ_{ci}	Hoek and Brown (1997)	$E_m = \sqrt{\frac{\sigma_{ci}}{100}}10^{\left(\frac{\text{GSI}-10}{40}\right)}$
	Hoek et al. (2002)	$E_m = (1 - 0.5D)\sqrt{\frac{\sigma_{\text{ci}}}{100}}10^{\left(\frac{\text{GSI}-10}{40}\right)}, \sigma_{\text{ci}} \leqslant 100\ \text{MPa}$
	Beiki et al. (2010)	$E_m = \tan\left(\sqrt{1.56 + (\ln(\text{GSI}))^2}\right)\sqrt[3]{\sigma_{ci}}$

quality tests and are commonly acknowledged as reliable data sources (Hoek and Diederichs, 2006). New simplified empirical equations are proposed by adopting Gaussian function to fit these *in situ* data. The proposed equations take the RMR classification system and the deformation modulus of intact rock (E_i) as input parameters. It has been demonstrated that the proposed equations fit well to the mentioned *in situ* data compared with the existing equations.

In this paper, the strategy of evaluation of existing equations for predicting E_m is described in Section 2. The performance of existing equations using the RMR and GSI classification systems is assessed in Section 3. The proposed simplified empirical relationships between E_m and the RMR system are described in Section 4.

2. The strategy of evaluation of existing empirical equations

2.1. Category

In this research, we focus only on the empirical equations which contain the RMR and GSI as input parameters. According to different input parameters, the existing empirical equations using the RMR and GSI classification systems can be divided into five groups (see Table 1).

2.2. Testing data

In situ data from Bieniawski (1978), Serafim and Pereira (1983) and Stephens and Banks (1989) are from high quality tests and are commonly acknowledged as reliable data sources (Hoek and Diederichs, 2006). These data also were widely used by many researchers (Barton, 1996; Palmström and Singh, 2001; Sonmez et al., 2006; Hoek and Diederichs, 2006) to assess the reliability of their proposed equations. Therefore, in this research, 43 of the 76 sets of these data were used for assessing the prediction performance of equations in Groups 1, 3 and 5. The other 33 sets of data which contain E_i as input parameter were used to test the prediction performance of equations in Groups 2 and 4.

These *in situ* data, however, are quantified on the basis of the RMR classification system. In order to use these data to evaluate the reliability of the empirical equations using the GSI system, the relationship between RMR and GSI will have to be used to transform RMR to GSI. Hoek and Diederichs (2006) suggested GSI equal to RMR if the RMR data were obtained before 1990. Therefore, for the *in situ* data which were collected before1989, the relationship of RMR = GSI is used in this research.

2.3. Indicators to assess the prediction performance of empirical equations

The value of Root Mean Square Errors (*RMSEs*) (Eq. (1)) and *R*-square (R^2) (Eq. (2)) are adopted in this research as indicators to assess the reliability of prediction by empirical equations:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(E_m^i - E_m^{i'}\right)^2} \tag{1}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(E_m^i - E_m^{i'}\right)^2}{\sum_{i=1}^{N}\left(E_m^i - E_m^{-}\right)^2} \tag{2}$$

where N is the number of testing data used, E_m^i and $E_m^{i\prime}$ are deformation modulus of rock masses obtained from the observed *in situ* data and derived from the empirical equations respectively. $\overline{E_m}$ is the mean value of E_m.

RMSE as defined is effectively the standard deviation of the errors associated with the estimation if it is unbiased. Clearly, the smaller the *RMSE*, the more reliable the estimation. The value of R^2 generally ranges from 0 to 1. For exact prediction, i.e., estimation with no error, the value of R^2 will be one. On the other hand, R^2 trends to zero for poor estimations. It should be noted that R^2 can be negative if the quality of the estimation is extremely poor.

3. The evaluation of existing empirical equations

3.1. Relations between E_m and RMR

Various attempts have been made to develop empirical equations taking the RMR as the input parameter to estimate E_m. These equations can be divided into two groups according to input variables as shown in Table 1.

3.1.1. Group 1 input parameter: RMR

The first empirical equation for predicting the rock mass deformation modulus using the RMR system was proposed by Bieniawski (1978), which was followed by other equations proposed by various