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Fractional description of mechanical property evolution of soft soils during creep

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Abstract: The motion of pore water directly influences mechanical properties of soils, which are variable during creep. Accurate description of the evolution of mechanical properties of soils can help to reveal the internal behavior of pore water. Based on the idea of using the fractional order to reflect mechanical properties of soils, a fractional creep model is proposed by introducing a variable-order fractional operator, and realized on a series of creep responses in soft soils. A comparative analysis illustrates that the evolution of mechanical properties, shown through the simulated results, exactly corresponds to the motion of pore water and the solid skeleton. This demonstrates that the proposed variable-order fractional model can be employed to characterize the evolution of mechanical properties of and the pore water motion in soft soils during creep. It is observed that the fractional order from the proposed model is related to the dissipation rate of pore water pressure.

Key words: *variable-order fractional model; fractional order; soil creep; evolution of mechanical properties; soft soil*

1 Introduction

The motion of pore water directly influences the mechanical properties of soft soils, which are composed of pore water and a solid skeleton. It has long been known that the mechanical properties of soft soils change during deformation or loading (Ferry 1980). However, until now, the relationship between the evolution of mechanical properties of soils and the motion of pore water is still unclear. The main reason is the lack of a suitable method to describe the change of soil mechanical properties. In hydraulic engineering and civil engineering, creep, which is the tendency of a solid material to move slowly or deform permanently under the influence of stresses, is the main mechanical process of soft soils. In this paper, we mainly focus on the description of the evolution of mechanical properties of soft soils during creep.

Fractional calculus has been considered one of the best mathematical tools for modeling physical responses and has been applied in a number of fields. The use of fractional calculus is

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motivated in large part by the fact that fewer parameters are required to achieve accurate approximation of experimental data. Previously, the creep response has been characterized primarily with the Maxwell, Kelvin-Voigt, and standard linear solid models (Ferry 1980) for the constitutive relationship. Bagley and Torvik (1983, 1985, 1986) and Koeller (1984) have developed models using fractional calculus. Other researchers (Padovan 1987; Shah and Qi 2010; Libertiaux and Pascon 2010; Lazopoulos 2006; Enelund et al. 1999; Enelund and Olsson 1999; Eldred et al. 1996; Gaul et al. 1991) have examined various issues involved in the numerical implementation of these sorts of models. Most of the fractional models mentioned above are called component models, and are based on a linear combination of elements, Hooke springs, and the fractional derivative Abel dashpot. Here, the fractional derivative Abel dashpot obeys the following expression:

$$\sigma(t) = E\theta^{\alpha} \frac{\mathrm{d}^{\alpha}\varepsilon(t)}{\mathrm{d}t^{\alpha}} \tag{1}$$

where σ and ε are the stress and strain, respectively; E and θ are material constants; t is time; and α is the fractional order, with $0 \le \alpha \le 1$. However, the component model is a mathematical model, and it is merely used to describe the mechanical response and does not consider the mechanical properties of materials. It is well known that the ideal solid obeys Hooke's law, $\sigma(t)\text{-}\varepsilon(t)$, and that Newtonian fluid satisfies Newton's law of viscosity, $\sigma(t)\text{-}\mathrm{d}\varepsilon(t)/\mathrm{d}t$. Thus, if we regard the mechanical properties as a spectrum, one end of which is pure elasticity, with $\alpha = 0$, then the other end is pure viscosity, with $\alpha = 1$. The fractional order of Eq. (1) can denote the location of a specific mechanical property on the spectrum, which can help us distinguish the mechanical property of materials quantitatively. However, we have found that some creep behaviors still cannot be simulated by Eq. (1). The primary reason is that the constant fractional order in Eq. (1) implicates the invariability of mechanical properties, while in the real world they change during the mechanical process. Therefore, representing the evolution of mechanical properties is a challenging issue in physical modeling and phenomenological description.

The concept of fractional order calculus needs to be further generalized by a calculus of varying order so that it is applicable to more complex mechanical properties of materials. Up to now, a number of variable-order fractional calculus definitions have been proposed (Coimbra 2003; Ingman and Suzdalnitsky 2004; Soon et al. 2005), and some of them have been applied to many fields such as anomalous diffusion (Sun et al. 2009; Umarov and Steinberg 2009), viscoelasticity (Ingman and Suzdalnitsky 2005; Ramirez and Coimbra 2007), multifractional Gaussian noises (Sheng et al. 2011), processing of geographical data (Cooper and Cowan 2004), and finite impulse response filters (Tseng 2006). However, variable-order calculus has not been used to describe the evolution of mechanical properties of soft soils during creep.

In our study, we attempted to describe the evolution of mechanical properties of soft soils