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Measures of association in contingency space analysis

Brian K. Martens[a,*], Lynne E. Gertz[a], Candace S. Werder[a], Jennifer L. Rymanowski[a], Karthik H. Shankar[b]

[a] Department of Psychology, Syracuse University, 430 Huntington Hall, Syracuse, NY 13244-2340, United States
[b] Center for Memory and Brain, Boston University, United States

HIGHLIGHTS

- The operant contingency value (OCV) has one variance term in the denominator.
- Phi and Yule's Q pool two and three variance terms, respectively.
- Yule's Q substantially overestimated both the OCV and phi with simulated data.
- The relationship between the OCV and phi was generally monotonic with empirical data.
- Unlike phi, the OCV was unaffected by discrepancies in diagonal cell frequencies.

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ABSTRACT

Sequential recording of behavior and its consequences is a common strategy for identifying potential maintaining variables in the natural environment. Disagreement remains over a standard approach to detecting contingent relations in the resulting data as well as a suitable association metric. In operant research, contingency is defined as the difference between the probability of reinforcement given some behavior and the probability of reinforcement given the absence of that behavior. Joint occurrences of behavior and its reinforcing consequences can be summarized in a 2 by 2 contingency table for which a variety of association measures exist. We analyzed three such measures algebraically (operant contingency value [OCV], phi coefficient, and Yule's Q), compared their relative magnitudes in a simulation study, and examined their relationship when computed on the same set of sequential observation data. Based on these analyses, we concluded that the OCV is a more robust measure for accurately indexing both absolute and relative degrees of contingency during functional behavior assessment.

1. Introduction

Functional behavior assessment (FBA) is a multi-step process involving the use of both indirect (e.g., interviews, rating scales) and direct (e.g., systematic observation) assessment methods to correlate socially significant problem behavior with events surrounding its occurrence in the natural environment (Gresham, Watson, & Skinner, 2001; Miltenberger, 2012; Witt, Daly, & Noell, 2000). Once the consequences associated with problem behavior have been identified (e.g., adult attention, escape from task demands), interventions can be designed to eliminate, reverse, or weaken those consequences to promote more desired appropriate behavior (Daly, Martens, Skinner, & Noell, 2009).

Although indirect FBA methods can be practical and efficient, the accuracy of information collected will be a function of caregivers' opportunities to observe problem behavior, limitations with recall, and potential biases stemming from problem behavior intensity (Martens & Lambert, in press). Direct FBA methods involve systematic observations of problem behavior at the time and place of its natural occurrence (Cone, 1977). Systematic observation provides more accurate descriptions of behavior and events surrounding its occurrence, and can be used to assess problem behavior and caregiver responses in a variety of settings (Martens, DiGennaro, Reed, Szczech, & Rosenthal, 2008). Despite its benefits, systematic observation requires some form of sequential recording (e.g., scoring occurrences of behavior and its immediate consequences) over multiple sessions, with between 50 and 300 min of observation time reported in most studies (Anderson & Long, 2002; Lalli, Browder, Mace, & Brown, 1993; Lerman & Iwata, 1993; Martens, Gertz, Werder, & Rymanowski, 2010; McKerchar & Thompson, 2004; Symons, Hoch, Dahl, & McComas, 2003). Moreover, once collected, the data must be analyzed to identify patterns in the delivery of caregiver responses and to generate hypotheses about potential sources of reinforcement based on these patterns

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[*] Corresponding author.
E-mail address: bkmarten@syr.edu (B.K. Martens).

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(e.g., positive reinforcement in the form of adult attention, negative reinforcement in the form of escape from task demands; Cataldo et al., 2012; Martens & Lambert, in press).

Currently, a standard approach to analyzing data from descriptive observations of problem behavior and its consequences as well as a suitable association metric are lacking in the FBA literature (Martens et al., 2008; McComas et al., 2009). Three different analytic strategies that have been reported to date include (a) computing the conditional probability of each consequence given the occurrence of problem behavior (Lalli et al., 1993; Mace & Lalli, 1991; Repp & Karsh, 1994); (b) comparing the conditional probability of a consequence given the occurrence of problem behavior to its base rate probability independent of behavior (McKerchar & Thompson, 2004; Vollmer, Borrero, Wright, Van Camp, & Lalli, 2001); and (c) comparing the conditional probability of a consequence given the occurrence of problem behavior to its conditional probability given the absence of problem behavior. This latter approach, known as contingency space analysis (CSA; Martens et al., 2008, 2010), can be used to identify the direction and magnitude of potential reinforcement effects from descriptive assessment data and is rooted in quantitative models of operant responding proposed by Estes and others (Estes, 1950a; Gibbon, Berryman, & Thompson, 1974).

1.1. Connection to Estes' research

A central tenet of Estes' (1950a) seminal paper was the view that an individual is "always doing something". For research purposes, this behavior stream could be divided into mutually exclusive and exhaustive response classes (R-classes), each with its own probability of occurrence, and with these probabilities summing to one (i.e., R-class and not-R-class responses). In discussing the theory of reinforcement schedules in basic operant research, Schoenfeld and Farmer (1970) used similar language, and suggested that defining an R-class automatically defines not-R-class responses, although the latter are infrequently acknowledged. Estes (1950a) suggested further that "learning and unlearning [involves]…transfers of probability relations between R-classes" (p. 97), and his linear operator model of probability matching in discrete-trial experiments was a precursor to Herrnstein's hyperbola of relative reinforcement matching in free-operant research (Gibbon et al., 1974; Myerson & Miezin, 1980). The matching law (Herrnstein, 1961) and its single-alternative version known as Herrnstein's law of effect (1970) predict that the relative rates of responding across two alternatives will match the relative rates of obtained reinforcement across those same alternatives. A considerable amount of basic and applied research on responding under concurrent variable-interval schedules has shown that choice behavior indeed approximates matching (e.g. Borrero et al., 2007; Borrero & Vollmer, 2002; Davison & McCarthy, 1988; Martens & Houk, 1989; Reed & Martens, 2008). Consistent with Estes' (1950b) view that the learning of one response involves the concurrent extinction of other responses, Staddon (1977) proposed that reinforcing one response has an inhibitory effect on all other responses, assuming a ceiling on response rate. That is, reinforcing a response (R1) decreases preference for the concurrently available alternative (R2), and the rate of decrease for R2 is proportional to the rate of reinforcement for R1 (Myerson & Miezin, 1980).

CSA is based on the same assumptions, but it compares the relative reinforcement rates (i.e., joint conditional probabilities of a consequence) for two mutually exclusive and exhaustive response classes. As an extension of Estes' early work on transfer of conditioning, CSA can be used to identify the degree of contingency and therefore the direction and magnitude of potential reinforcement effects in natural environments. The goals of this paper are to define contingency from an operant perspective, analyze three statistics that are commonly used to index the degree of association in 2 by 2 tables, and consider which might be preferable for summarizing behavior–consequence relations. In order to address this latter goal, we examine the relationship between the operant contingency value (hereafter referred to as the OCV), the phi coefficient, and Yule's Q in a simulation study and when computed on the same set of sequential observation data.

	Y	~Y	
X	a (10)	b (10)	20
~X	c (20)	d (60)	80
	30	70	100

Fig. 1. A typical 2 by 2 contingency table relating occurrences of a consequence (Y) to behavior (X). For the sample data (given within brackets), OCV $= 0.25$, phi $=$ 0.22, and Yule's $Q = 0.50$.

1.2. Operant definition of contingency

Contingency is defined in operant research as the difference between the probability of reinforcement given some behavior and the probability of reinforcement given the absence of that behavior (Hammond, 1980; Mathews, Shimoff, & Catania, 1987). It is widely acknowledged that operant conditioning requires a contingency between response and reinforcer (Gibbon et al., 1974; Schwartz, 1989; Vollmer et al., 2001). In the absence of such a contingency, reinforcement effects do not occur, and previous levels of behavior maintained by contingent reinforcement decrease (e.g., Goh, Iwata, & DeLeon, 2000; Hammond, 1980).

If we consider the occurrence of behavior and its reinforcing consequence as two dichotomous events (i.e., present/absent), then four joint outcomes are possible; (a) both behavior and the consequence occurred, (b) behavior occurred but was not followed by the consequence, (c) behavior did not occur but the consequence was still delivered, and (d) neither behavior nor the consequence occurred. The joint outcomes of two dichotomous events are typically summarized in a 2 by 2 table similar to the one shown in Fig. 1, where X stands for behavior, $\sim X$ stands for the absence of behavior, Y stands for the consequence, and $\sim Y$ stands for the absence of the consequence (Bakeman, McArthur, & Quera, 1996). Individual cell values show the frequency with which each *joint* outcome was observed, whereas marginal values show the frequency of X and Y occurring *independent* of the other event (probabilities may be used also).

From the table presented in Fig. 1, the probability of a consequence given behavior is computed as $a/(a+b)$, and it can be taken as an approximation of the schedule on which the consequence follows behavior. The probability of a consequence given the absence of behavior is computed as $c/(c + d)$, and it indicates the schedule on which that same consequence is delivered for behavior other than the target. CSA involves plotting these joint conditional probabilities in coordinate space, and it can be used to identify events contingent on problem behavior prior to treatment to identify potential maintaining variables, evaluate changes in reinforcement delivery following treatment, and assess the fidelity with which treatment is implemented across sessions (see Martens et al., 2008, for a detailed discussion). Put another way, CSA can be