Journal of Mathematical Psychology 58 (2014) 21–32

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

Survivor interaction contrast wiggle predictions of parallel and serial models for an arbitrary number of processes

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HIGHLIGHTS

- We explore the precise behavior of the serial exhaustive SIC function for $n = 2$.
- We provide a generalization of the SIC function to an arbitrary number of processes.
- We analyze the generalized SIC for both parallel and serial models with minimum and maximum time stopping rules.
- We demonstrate application of the theorems to data from a short-term memory search task.

ARTICLE INFO

Article history:
Available online 21 December 2013

Keywords:
Survivor interaction contrast
Human information processing
Logarithmic concavity
Multi-processes

ABSTRACT

The Survivor Interaction Contrast (SIC) is a distribution-free measure for assessing the fundamental properties of human information processing such as architecture (i.e., serial or parallel) and stopping rule (i.e., minimum time or maximum time). Despite its demonstrated utility, there are some vital gaps in our knowledge: first, the shape of the serial maximum time SIC is theoretically unclear, although the one 0-crossing negative-to-positive signature has been found repeatedly in the simulations. Second, the theories of SIC have been restricted to two-process cases, which restrict the applications to a limited class of models and data sets. In this paper, we first prove that in the two-process case, a mild condition known as strictly log-concavity is sufficient as a guarantor of a single 0-crossing of the serial maximum time SIC. We then extend the definition of SIC to an arbitrary number of processes, and develop implicated methodology of SIC in its generalized form, again in a distribution-free manner, for both parallel and serial models in conjunction with both the minimum time and maximum time stopping rules. We conclude the paper by demonstrating application of the theorems to data from a short-term memory search task.

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1. Introduction

The question of whether people can perform multiple perceptual or mental operations simultaneously, that is, parallel processing, vs. whether items or tasks must proceed serially (one at a time), has intrigued psychologists since the birth of experimental psychology. Historically, reaction time (RT) has been the primary measure on this question. The work of the physiologist F.C. Donders (e.g., Donders, 1868) was seminal in this regard, although other researchers, such as W. Wundt, were more prolific with regard to early results on human cognition.

With the revolution brought about through cognitive science and cognitive psychology in the 1950s and 1960s, questions such as the parallel vs. serial conundrum, which had lain dormant since the nineteenth century saw a renaissance of interest.

The serial vs. parallel topic is our primary concern here. However, it may be worth a moment's pondering, given the pioneering role of William K. Estes in the advent of mathematical psychology, of how the latter field, and Estes' research, fit into, and contributed to, modern cognitive psychology. Three tributaries fed the new stream of mathematical psychology in the 1950s and 60s. These were: 1. Signal detection theory, child of psychophysics and sensory processes, mathematical communications theory, applied physics, and statistical decision making (e.g., Green & Swets, 1966; Tanner & Swets, 1954). 2. Foundational measurement the offspring of S.S. Stevens' brilliant but non-rigorous statements concerning measurement in psychology fostered and rendered rigorous through strands from philosophy, mathematical logic and abstract algebra (e.g., Krantz, Luce, Suppes, & Tversky, 1971; Roberts & Zinnes, 1963). 3. Mathematical learning theory which went back at least to Clark Hull (e.g., Hull, 1952); or see Koch's elegant summary in Modern Learning Theory (Koch, 1954). This branch is where we find the Estes trailblazing Stimulus Sampling Theory (Estes, 1955, 1959), a precise, quantitative theory of human and animal learning.

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0022-2496/$ – see front matter. Published by Elsevier Inc.
http://dx.doi.org/10.1016/j.jmp.2013.12.001

This theory, which still impacts a wide spectrum of research in cognition today, led to a score of research advances by Estes and colleagues as well as a host of other scientists (e.g., Atkinson & Estes, 1963; Friedman et al., 1964).

Estes was an early entrant into the embryonic cognitive movement. His research in this domain was likely influenced by the burgeoning efforts utilizing the information processing approach, perhaps the early dominant theme in this new domain. Early pioneers included Wendell Garner (e.g., Garner, 1962), Donald Broadbent (e.g., Broadbent, 1958), William Hick (e.g. Hick, 1952), and Colin Cherry (e.g. Cherry, 1953) (note the heavy presence of British psychologists).

American psychologists were soon contributing to this rapidly expanding field which bridged sensory processes, higher perception, and elementary cognition. Prime examples are Charles Eriksen (e.g., Eriksen & Spencer, 1969), Michael Posner (e.g., Posner, 1978), Raymond Nickerson (e.g., Nickerson, 1972), Ralph Haber (e.g., Haber & Hershenson, 1973), and Howard Egeth (e.g., Egeth, 1966). And, Bill Estes of course.

The employment of ingenious experimental designs to answer questions concerning whether humans perform visual or memory search in a serial or parallel fashion provide apt examples of new trends making an appearance in the 60s and 70s. (e.g., Sperling, 1960, 1967; Sternberg, 1966, 1975). Estes and colleagues provided some classic early results in this domain in extending, and mathematically modeling extensions of Sperlings innovative visual search experimental designs. For instance, Estes and Taylor (1964) developed a new detection method as well as associated models in this vein. Also, Estes and Taylor (1966) and Estes and Wessel (1966) were beginning to explore phenomena and human information processing mechanisms related to the presence of redundant signals in visual displays.

The Sternberg (1966) innovative and rather startling RT data in short term memory search, in particular, had a profound influence on thinking in the parallel vs. serial processing literature. In fact, a massive body of experimental literature over several decades has been based on the inference that increasing, more-or-less straight-line RT functions of the workload n,[1] the number of comparisons to perform, imply serial processing. However, the ability of limited capacity parallel models to mimic serial models, in the strong sense of mathematical equivalence, was demonstrated relatively early on (e.g., Atkinson, Holmgren, & Juola, 1969; Murdock, 1971; Townsend, 1969, 1971).[2] And in fact, the reverse possibility of serial models to mimic parallel models was also proven (Townsend, 1969, 1971, 1972, 1974). The early mathematical results were confined to limited types of RT distributions, but later developments extended to arbitrary probability distributions (Townsend, 1976; Townsend & Ashby, 1983; Vorberg, 1977).

The parallel models which perfectly mimic serial models are *limited capacity* in the sense that their processes degrade in their efficiency as the workload n increases. Such models intuitively make the predictions associated with serial processing, specifically the linear RT graphs of the workload n (e.g., Townsend, 1971). Fortunately, theory-driven experimental methodologies have been invented in recent years that are considerably more robust in the assessment of mental architecture, particularly serial vs. parallel processing (Scharff, Palmer, & Moore, 2011; Townsend, 1976, 1981, 1990a; Townsend & Nozawa, 1995; Townsend & Wenger, 2004). In particular, the new methodologies often allow architectural inferences even though the workload is held constant, so that capacity does not confound architectural inferences.

Our focus here lies within the general approach referred to as *Systems Factorial Technology* (hereafter SFT; see Townsend, 1992; Townsend & Nozawa, 1995). A number of investigators have made essential contributions to this literature including Schweickert and Dzhafarov and colleagues (Dzhafarov, 1997; Dzhafarov, Schweickert, & Sung, 2004; Schweickert, 1978, 1982; Schweickert & Giorgini, 1999; Schweickert, Giorgini, & Dzhafarov, 2000). SFT relies heavily on mathematical propositions indicating experimental conditions where strong tests of architectures may be found, although other testable features, such as capacity, are also encompassed presently. The bulk of theoretical work has been performed under the assumption of *selective influence*. Our scope prohibits details here, but we can loosely define selective influence as the property that certain experimental factors act only on specific processes in the overall system (see Section 2.1 for more detailed discussion on selective influence). When selective influence is in force, predictions of serial and parallel models and the pertinent decisional stopping rules are strikingly distinct. This paper is intended to significantly strengthen and extend these predictions.

SFT requires the survivor function $S(t)$, which is simply the complement of the well-known cumulative distribution (or frequency) function (the CDF) written as $F(t)$. That is, $S(t) = 1-F(t)$. A central statistical diagnostic is then the survivor interaction contrast (or SIC) function. It performs a double difference contrast operation on the survivor functions that is analogous to the mean interaction contrast (or MIC) employed on the arithmetic RT means in earlier investigations (e.g., Schweickert, 1978; Sternberg, 1966). However, it now expresses a highly diagnostic function of time, rather than a single number.

Despite the successful deployment of the SIC measure, there are some vital gaps in our knowledge, restricting the applications to a limited class of models and data sets. These will be sketched within a brief presentation of relevant knowledge we do have.

We know that, for $n = 2$, serial minimum time models predict perfectly flat signatures whereas serial maximum time (i.e., the classical exhaustive processing time stopping rule; see Sternberg, 1969; Townsend, 1974) predictions must include at least one wiggle (i.e., the up-and-down excursions marked by 0-crossings) below and above 0 (Townsend & Nozawa, 1995). However, although simulations have intimated that there is a single wiggle passing through 0, in a negative-to-positive direction as exhibited in Fig. 1 (the top right panel), this has not been shown to be true for all distributions. In fact, the exact shape of the SIC curve is as yet unknown.

Therefore, in elucidating further properties of serial exhaustive processing: A. We first prove that serial exhaustive processing inevitably predicts an odd number of 0-crossings in the $n = 2$ case. B. Next, we show that a certain readily-met mathematical condition is sufficient to force the behavior indicated through our simulations, a single 0-crossing of the SIC function.

The behavior our SIC signatures have also remained unidentified for $n > 2$, in the case of all studied serial and parallel processes up to now. The quite intriguing behaviors in the case of the serial and parallel models with varying stopping rules, and for arbitrary values of n, are next developed for: A. Serial minimum time processing. B. Serial maximum time processing. C. Parallel minimum time processing. D. Parallel maximum time processing.

Successful completion of the above goals should significantly expand the possibilities of application. Since mathematical details of SFT in general, have been published elsewhere (e.g., Townsend & Nozawa, 1995) and tutorials are available (e.g., Townsend, Fific, & Neufeld, 2007; Townsend & Wenger, 2004; Townsend et al., 2011), only the bare bones SFT can be displayed here.

[1] An increment in workload is usually natural to define in terms of number of dimensions, or subtasks involved in some task. We shall often refer simply to items or, sometimes, processes as generic tags for the discrete objects being processed or the conduits working on them. The unit of workload typically relates in a natural fashion to the task. For example, if a memory search task involves examination of a list of letters, the unit may be made straightforwardly in terms of letters. Then n may stand for both the workload in the task and the number of letters in the memory set.

[2] For an up to date review of the parallel–serial identifiability issue, see Townsend, Yang, and Burns (2011).