Two topics in Tree Inference: Locating a phonological network effect in immediate recall and arborescence partitive set form

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HIGHLIGHTS

- We show how the form of a multinomial processing tree can be inferred from data.
- A tree is inferred from data in the literature on recall from a phonological network.
- A comparability graph can combine results from different experiments to infer a tree.
- Uniqueness of the tree depends on partitive sets.
- We describe the form of partitive sets in multinomial processing trees.

ARTICLE INFO

ABSTRACT

In a multinomial processing tree, processes are represented by vertices in an arborescence, i. e., a rooted tree with arcs directed away from the root. Processing begins at the root. When a process is completed one of its possible outcomes is produced. This outcome may result in another process starting, or in a response at a terminal vertex. The form of a multinomial processing tree can sometimes be inferred from data. Suppose an experimental factor, e.g., an item's serial position, changes probabilities of outcomes of a single vertex in the tree, all else invariant. The factor is said to selectively influence the vertex. Suppose each of two factors in an experiment selectively influences a different vertex in a multinomial processing tree. The tree is equivalent to one of two relatively simple trees and the data will indicate which applies. As an example we construct a multinomial processing tree from data of Vitevitch et al. (2012) on immediate serial recall. One factor was the clustering coefficient in a phonological network of a to-be-remembered word. The other was the word's serial position. The clustering coefficient selectively influences a vertex that precedes a vertex selectively influenced by serial position. The clustering coefficient has two opposing effects on recall. Part two of the paper is about a technical question. Suppose we learned for every pair of processes in a task whether they are executed in order or not. A comparability graph can represent the information. If there is an underlying arborescence it can be constructed with a procedure in the literature, the Transitive Orientation Algorithm. More than one arborescence may be possible. Different possibilities arise from subsets of vertices called partitive sets. We show that a partitive set in an arborescence has a simple form. Vertices in commonly used Multinomial Processing Trees can be ordered in such a tree in only one way.

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1. Introduction

Multinomial processing trees are successful models of many cognitive phenomena. Topics include perception (Ashby, Prinzmetal, Ivry, & Maddox, 1996), memory (Batchelder & Riefer, 1986; Chechile & Meyer, 1976; Xi, 2014), and social cognition (Klauer & Wegner, 1998). For reviews, see Batchelder and Riefer (1999) and Erdfelder et al. (2009).

Typically, a tree model is proposed based on intuition and experience, and evaluated by its goodness of fit to data. Here we discuss an alternative, Tree Inference, for starting with data from a factorial experiment and constructing a suitable tree (Schweickert & Chen, 2008; Schweickert & Han, in preparation; Schweickert & Xi, 2011). We demonstrate by constructing a multinomial processing tree for data on immediate serial recall by
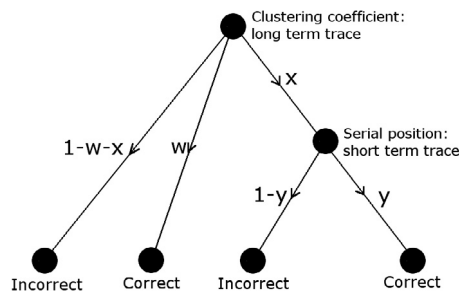
Fig. 1. Multinomial processing tree inferred from data of Vitevitch et al. (2012) Experiment 3. This is the Standard Tree for Ordered Processes. Labels about memory traces are specific for the experiment.

Vitevitch, Chan, and Roodenrys (2012). We then consider an issue that arises when constructing a tree by combining information about pairs of vertices with a procedure called the Transitive Orientation Algorithm. For those interested in technical details at this point, the uniqueness of a tree constructed from information about which pairs of vertices are on a path together and which are not depends on certain subsets of vertices called partitive sets (or modules). We show how the parts of a partitive set are arranged in a directed tree.

Fig. 1 shows a multinomial processing tree for data of Experiment 3 of Vitevitch et al. (2012). In such a tree, every vertex represents a process that has a number of mutually exclusive possible outcomes. A process in a recall experiment might be, for example, an attempt to retrieve an item from memory, and the possible outcomes might be success or failure. If there is a path from a vertex u to another vertex v (with every arc followed in the direction indicated by its arrow), then u precedes v. If vertex u precedes vertex v or vertex v precedes vertex u, then u and v are ordered; otherwise they are unordered. Processing in the task starts at the source, a vertex preceded by no other vertex. Possible outcomes of a process are represented by arcs descending from the vertex representing the process. Associated with each arc is the probability that the corresponding outcome occurs. In Fig. 1, probabilities are denoted by expressions such as x, y and $1 - y$. Each arc ends at a vertex that represents either a process that starts when the outcome represented by the arc occurs, or a response made when the outcome occurs. A vertex representing a response is called a terminal vertex (or sink), and is followed by no other vertex. Each terminal vertex is associated with a response class, for example, correct or incorrect. The probability of a response at a particular terminal vertex is the product of the probabilities associated with the arcs on the path from the source to the terminal vertex. The probability of a response in a certain class is the sum of the probabilities of responses at all the terminal vertices associated with that class.

This paper has two distinct parts. In the first we show by example how results from a single experiment with two factors can determine the form of a multinomial processing tree accounting for the data. In the second we consider a technical question that arises when considering how information from several such experiments could be combined to form a single multinomial processing tree. More than one such tree may be possible. Schweickert and Han (2016) showed that more than one is possible if and only if there are two or more vertices in series, as we will explain here later. Vertices in series are a special case of sets of vertices called partitive sets (e.g., Golumbic, 1980), and we show here that the form of a partitive set is relatively simple.

The inquiry in the second part of the paper arises by noticing that trees have now been constructed from several experiments on immediate serial recall, each experiment manipulating two or three factors (see Schweickert, Fisher, & Sung, 2012 for examples). Looking ahead, suppose results from small experiments

accumulate to the point where we have data for every pair of processes involved in a task. For each pair, suppose at least one multinomial processing tree has been constructed, and we know whether the vertices representing the pair are ordered or unordered, the trees being consistent about this. It may be possible to represent the order information of the separate experiments in a single multinomial processing tree. If one is possible, it can be constructed with the Transitive Orientation Algorithm, a procedure developed in graph theory (see, e.g., Golumbic, 1980). Although the procedure is well known, questions remain. More than one tree may be possible. The possibilities are determined by certain sets of vertices called partitive sets. We explain when the tree is unique, and consider the form of partitive sets in a directed rooted tree. The two parts of the paper are about the best understood situations, when information is known about one pair of vertices, from one two factor experiment, and when it is known about all pairs, from many two factor experiments. The second part of the paper is unavoidably technical.

We turn now to explaining how a multinomial processing tree can be constructed in one of the well understood situations, through an experiment that manipulates two factors. As mentioned earlier, there are two ways two distinct vertices can be arranged in a multinomial processing tree. If one vertex precedes the other, the vertices are ordered (or comparable). If neither precedes the other, the vertices are unordered (or incomparable). The problem of distinguishing ordered and unordered processes in a multinomial processing tree is analogous to that of distinguishing serial and parallel processes (Townsend, 1972); for reviews see Logan (2002) and Townsend and Wenger (2004).

We can discover how two vertices are arranged by finding experimental factors that selectively influence the processes the vertices represent. Sternberg (1969) pioneered the use of selectively influencing processes to analyze reaction times. Effects on accuracy of selectively influencing processes were investigated by Schweickert (1985), Jacoby (1991) and Hu (2001). The difficult problem of selectively influencing processes that are stochastically interdependent was investigated by Townsend (1984) and Dzhafarov (2003); see Dzhafarov and Kujala (2014) for recent discussion.

Consider a task performed by executing processes in a multinomial processing tree. Suppose changing the level of an experimental factor changes probability values associated with arcs descending from a single vertex, leaving all else invariant. The factor is said to selectively influence the process represented by the vertex. (Factors changing probabilities on arcs descending from more than one vertex are discussed by Schweickert and Xi (2011), but are beyond the scope of this paper.) If two factors selectively influence two different processes, each represented by a vertex, the data will satisfy certain conditions explained below (following Eqs. (1) and (2) in Section 2). The conditions differ depending on whether the vertices representing the two processes are ordered or unordered, and the conditions determine both the form of a multinomial processing tree and values of its parameters. There are only three possibilities. If the conditions required for ordered vertices are met, a multinomial processing tree of the form illustrated in Fig. 1 exists, in which ordered vertices represent the processes selectively influenced by the factors. If the conditions required for unordered vertices are met, a multinomial processing tree of the form illustrated in Fig. 2 exists, in which unordered vertices represent the processes selectively influenced by the factors. If the conditions are not satisfied, no multinomial processing tree exists in which the factors selectively influence different processes, each represented by a vertex (Schweickert & Chen, 2008).

When factors selectively influence processes, a psychological interpretation of the processes can often be based on what the