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Half-full or half-empty? A model of decision making under risk

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HIGHLIGHTS

- A new descriptive model of decision-making under risk.
- Retains much of the intuitive appeal of the expected value model.
- Explains various paradoxes in decision-making under risk.
- Relies on only two parameters that have clear behavioral interpretations.

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ABSTRACT

We propose a descriptive model of decision making under risk, inspired by the "half-full, half-empty" glass metaphor, that explains well-known paradoxes identified by Allais (1953), Kahneman and Tversky (1979), and Birnbaum (2008). The model is intuitive in that it is closely related to the expected value criterion and its parameters have a clear behavioral interpretation, and parsimonious in that it provides an approach to modeling behavior based on only two parameters.

1. Introduction

The development of the literature on decision-making under risk has been driven by the discovery of paradoxes afflicting existing theories (Fox, Erner, & Walters, in press, Luce, Ng, Marley, & Aczél, 2008, Wu, Zhang, & Gonzalez, 2004). Old paradoxes such as the St. Petersburg paradox (Bernoulli, 1738) and the Allais (1953) paradox and its variations (Kahneman & Tversky, 1979) challenge the descriptive power of the expected value (EV) criterion and expected utility (EU) theory. It is for this reason that, over the last 60 years or so, decision theorists have sought a theory of choice able to provide a satisfactory description of decision-makers' behavior in risky situations.

While numerous theories have been proposed (reviewed by Fox et al., in press, Starmer, 2000, Wu et al., 2004), Kahneman and Tversky's (1979) original prospect theory and Tversky and Kahneman's (1992) new prospect theory have become the leading descriptive framework for modeling decision making under risk (Camerer, 1998, Fox & Poldrack, 2014) and have inspired a large body of theoretical and empirical work.[1] However, despite this apparent success, various studies have questioned the underlying assumptions and applicability of prospect theory (PT) on the basis of empirical violations it seems unable to accommodate.[2] These violations include forms of description invariance, context independence, internality, gain–loss separability, coalescing, and prob-

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[1] See, for instance, Abdellaoui (2000, 2002), Camerer (1992), Gonzalez and Wu (1999), Karni and Safra (1987), Luce (2000, 2001), Machina (1982), Prelec (1998), Quiggin (1993), Schmeidler (1989), Starmer and Sugden (1989), Tversky and Wakker (1995), Wakker (1994, 1996, 2001), Yaari (1987).

[2] See, for instance, Baltussen, Post, and van Vliet (2006), Brandstätter, Gigerenzer, and Hertwig (2006), Hertwig, Barron, Weber, and Erev (2004), Humphrey (1995), Lopes and Oden (1999), Marley and Luce (2005), Neilson and Stowe (2002), Payne (2005), Starmer (2000), Starmer and Sugden (1993), Wu and Markle (2008), Wu, Zhang, and Abdellaoui (2005).

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ability outcome independence (Fox et al., in press). PT is consequently at once one of the most confirmed and falsified decision making models currently out there (Wakker, 2010, p. 351).

Although there is a debate about whether some of these studies provide a definite falsification of PT,[3] Birnbaum (2008) has recently produced a systematic empirical study that has received considerable attention in the decision making literature (Fox et al., in press, Luce et al., 2008, Wakker, 2010). In particular, he presents 11 new paradoxes that lead him to conclude that PT cannot "be retained as a descriptive model of decision making" (Birnbaum, 2008, p. 464).

Various "configural weight models" have been devised that accommodate some of these paradoxes, including the rank-affected multiplicative weights (RAM) model (Birnbaum, 1997), the transfer of attention exchange (TAX) model (Birnbaum & Chavez, 1997, Birnbaum & Stegner, 1979), and the gains decomposition utility (GDU) model (Luce, 2000, Marley & Luce, 2001, Marley & Luce, 2005). However, while these models fit the data better than PT in many experiments (Birnbaum, 2008, Fox et al., in press, Wakker, 2010) and are more flexible as they usually have more free parameters, they are less parsimonious (Wu et al., 2004, p. 408) and "do not have the particularly tractable form of prospect theory, with psychological interpretations for its parameters…" (Wakker, 2010, p. 351).

The present paper accordingly aims to provide an intuitive and parsimonious descriptive model of decision making under risk that can explain well-known "old and new" paradoxes at one stroke. The intuition underlying the model can be easily grasped by way of the "glass half-full, glass half-empty" metaphor, which suggests that different individuals may evaluate the same situation in different ways, and will typically be divided between those who focus on positive aspects of the situation (the optimists) and those who focus on negative aspects (the pessimists). In the same spirit, we suggest that a decision maker's (DM) assessment of a choice situation may depend on the way she values the difference between the outcomes above and below the mean value of the lottery in question. The lottery's evaluation (and consequently the DM's risk attitude) depends only on two parameters: her degree of optimism/pessimism λ, and the probability distortion parameter q that determines her decision weights. We will show that λ and q can be calibrated such that old and new paradoxes are explained.

When compared to other models of decision-making recently proposed to accommodate Birnbaum's (2008) paradoxes, our model, which we call the Half-Full/Half-Empty (henceforth HFHE) model, has two important advantages. The first is that it is highly intuitive, both for being closely related to the EV criterion and for its parameters having a clear and widely used behavioral interpretation (see, for instance, Arrow & Hurwicz, 1972, Hurwicz, 1951, Kahneman & Tversky, 1979, Tversky & Wakker, 1995). The second is that it is highly parsimonious in that it provides a very simple way of modeling behavior based on only two parameters.

We begin by introducing the HFHE model and discussing its main properties. To facilitate the presentation, we develop the model with reference to the Allais (1953) paradox and explain how this paradox can be accommodated. We then show how the HFHE model can provide a solution to additional paradoxes presented in Birnbaum (2008) and Kahneman and Tversky (1979). We close with a short conclusion.

2. The HFHE model

The glass half-full/half-empty adage suggests that, when observing a glass that contains 50% of wine, optimists focus on the half-full part and therefore see the glass half-full, while pessimists focus on the half-empty part and therefore see the glass half-empty. This is the intuition we pursue here, namely that the DM's assessment of a given lottery depends on the way she evaluates the difference between the outcomes above and below the expected value of the lottery.

Too see this, consider a lottery X with finitely many non-negative outcomes $X = (p_1 : x_1, p_2 : x_2, \dots, p_n : x_n)$ with $x_i \geq 0$ for $i = 1, \dots, n$. We can express the EV representing function for the lottery X as:

$$H_{EV}(X) = \mathbb{E}[X] = \mu + \mathbb{E}[X - \mu] \quad (1)$$

where $\mu = \mathbb{E}[X]$ is the expected value of X. Since $y = (y)_+ + (y)_-$, where $(y)_+ = \max\{y, 0\}$ and $(y)_- = \min\{y, 0\}$, Eq. (1) can be rewritten as:

$$H_{EV}(X) = \mu + 2\left\{\frac{1}{2}\mathbb{E}\left[(X-\mu)_+\right] + \frac{1}{2}\mathbb{E}\left[(X-\mu)_-\right]\right\}. \quad (2)$$

The EV representing function H_{EV} in Eq. (2) assigns an equal weight to $\mathbb{E}\left[(X-\mu)_+\right]$ and $\mathbb{E}\left[(X-\mu)_-\right]$. However, in line with the half-full/half-empty glass metaphor, suppose that the DM assigns different weights according to her degree of optimism/pessimism (see, for instance, Arrow & Hurwicz, 1972, Hurwicz, 1951). In particular, suppose that an optimist who sees the glass half-full, overweighs the "large" outcomes ($x_i > \mu$) relative to the "small" ones ($x_i < \mu$), while a pessimist, who sees the glass half-empty, does the opposite. To take into account this behavioral feature, we introduce into the model a parameter λ representing the DM's degree of optimism/pessimism, and propose the following simple generalization of the EV representing function in Eq. (2):

$$H_\lambda(X) = \mu + 2\left\{\lambda\mathbb{E}\left[(X-\mu)_+\right] + (1-\lambda)\,\mathbb{E}\left[(X-\mu)_-\right]\right\} \quad \text{with } 0 \leq \lambda \leq 1. \quad (3)$$

If the DM is an optimist, $\frac{1}{2} < \lambda \leq 1$ and she overweighs $\mathbb{E}\left[(X-\mu)_+\right]$ and underweights $\mathbb{E}\left[(X-\mu)_-\right]$. If the DM is a pessimist, $0 \leq \lambda < \frac{1}{2}$ and the opposite occurs. If the DM is neither an optimist nor a pessimist, $\lambda = \frac{1}{2}$ and the EV representing function H_{EV} in Eq. (2) is recovered.

Example 1. *Explanation of the Allais paradox.* Allais (1953) introduced the most famous counter-example to EU theory showing that people overweight certain outcomes relative to probable ones (the so-called certainty effect). The representing function H_λ that we have introduced in Eq. (3) is very simple, yet able to describe the original Allais (1953) paradox represented in Table 1. The notation used is (outcome, corresponding probability) and monetary outcomes are in French francs.

Table 1
The Allais (1953) choice problems.

Problem 1		Problem 2	
A	B	C	D
(100 million, 1)	(500 million, 0.10) (100 million, 0.89) (0, 0.01)	(100 million, 0.11) (0, 0.89)	(500 million, 0.10) (0, 0.90)

When facing Problem 1 and Problem 2, according to EU theory, a DM should choose either A and C or B and D. The paradox arises because when presented with a choice between A and B most DMs select A, while when presented with a choice between C and D, most DMs select D. A DM characterized by the representing function in Eq. (3) prefers A to B when $0 \leq \lambda < 0.23$ and prefers D to C when $0.22 < \lambda \leq 1$. Hence for $0.22 < \lambda < 0.23$ the representing function H_λ is able to explain the Allais (1953) original paradox. ■

[3] See, for instance, Baucells and Heukamp (2006), Birnbaum and McIntosh (1996), Camerer and Ho (1994), Fox and Hadar (2006), Rieger and Wang (2008), Wakker (2003), Wu and Gonzalez (1996), Wu et al. (2004).