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Cultural consensus theory for continuous responses: A latent appraisal model for information pooling

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HIGHLIGHTS

- A new Cultural Consensus Theory (CCT) model for continuous data.
- Model-based clustering and detection of multiple cultures in the data.
- Derivation of the mathematical and statistical properties of the model.
- Hierarchical Bayesian inference for the model on real and simulated data.
- User-friendly software that facilitates application of the model.

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ABSTRACT

A Cultural Consensus Theory approach for continuous responses is developed, leading to a new model called the Continuous Response Model (CRM). It is a cognitive psychometric model that is applicable to consensus data, in which respondents (informants) have answered questions (items) regarding a shared knowledge or belief domain, and where a consensus (a latent set of 'true' answers applicable to the group) may exist. The model estimates the consensus answers to items, item difficulty, informant knowledge and response biases. The model can handle subcultures that have different consensuses from one another in the data, and can both detect and cluster respondents into these subcultures; it thus provides one of the first forms of model-based clustering for multicultural consensus data of the continuous response type. The model is demonstrated on both simulated and real multi-cultural data, using the hierarchical Bayesian framework for inference; two posterior predictive checks are developed to verify the central assumptions of the model; and software is provided to facilitate the application of the model by other researchers.

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1. Introduction

The purpose of the present paper is to introduce a Cultural Consensus Theory (CCT) model for continuous responses, and in tandem, supply user-friendly software that facilitates application of the model by others. CCT is a methodology conceived of in the mid 1980s (Batchelder & Romney, 1986, 1988; Romney, Weller, & Batchelder, 1986) that is applicable to *consensus data*: defined as data in which respondents (informants) have answered questions (items) regarding a shared knowledge or belief domain, and where a consensus (a set of 'true' answers applicable to the group, or culture) may exist. Exemplary forms of consensus data may consist of eyewitness testimony, probability forecasting, political polls, cultural beliefs, subjective assessment, or ideological beliefs. In order to estimate the consensus answers to items, as well as item response effects (e.g. knowledge level, response biases, item difficulty, and cultural membership) for these forms of data, CCT consists of a number of cognitive psychometric models.

The first CCT model was developed for binary data (e.g. true/false data); it is called the General Condorcet Model (GCM), and makes the assumption that the consensus truth of each item is also a binary value. This model has been widely applied in the social and behavioral sciences, especially cultural anthropology (e.g. Weller, 2007). The detection of multiple cultural truths (subcultures with differing consensuses), and cultural membership for the GCM, was developed by Anders and Batchelder (2012). An alternate assumption to that of the GCM, that continuous (fuzzy) truths in (0, 1) instead underlie binary data, was explored by Batchelder and Anders (2012). They introduced a new model for binary data that used a beta appraisal distribution to estimate these values in (0, 1); the Latent Truth Model (LTM), and these values could represent such

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things as a degree of truth, intensity, or a true probability. Carrying on with this assumption, Anders and Batchelder (2013) developed an extensive CCT model for ordinal data (ordered discrete options of $C \geq 2$ categories) using a Gaussian appraisal model, which links the truth locations in (0, 1) to the real line,[1] called the Latent Truth Rater Model (LTRM). The model developed herein shares much similarity to the work of the LTRM, which can likewise detect and estimate multiple cultures/clusters of informants, continuous truths, shifting and scaling response biases, informant knowledge, and item difficulty for each culture, but it is instead specified for continuous data.

The Continuous Response Model (CRM), with its application methodology and software introduced here, offers to our knowledge, the first cognitive model-based approach for cultural consensus clustering for continuous response data. The model supposes that the continuous responses of the consensus data arise from the possible effects of membership to distinct cultural consensuses, consensus knowledge, item difficulties, and cognitive response biases. A natural interval to collect continuous responses is in (0, 1); the model acts on the logit transforms of these data, to utilize the benefits that have been found with using the Gaussian modeling approach over the beta approach, as in the work of the LTRM.[2] There have been several previous efforts to develop a CCT model for continuous response data (Batchelder & Romney, 1989; Batchelder, Strashny, & Romney, 2010; France & Batchelder, 2014). However, the current model goes well beyond these efforts by extending the specification of the model to include multicultural detection, model-based clustering, and a fully hierarchical inference structure complete with Bayesian posterior predictive checks of crucial assumptions of the model. A software package that greatly facilitates its application, has also been introduced with the model.

There has also been recent progress for modeling continuous responses outside of CCT (Merkle & Steyvers, 2011; Steyvers, Wallsten, Merkle, & Turner, 2013; Turner & Steyvers, 2011; Turner, Steyvers, Merkle, Budescu, & Wallsten, 2013a), and these models have been primarily introduced for the aim of forecasting aggregation, rather than cultural consensus work. Each of these models shares some characteristics with the CRM, but may differ in regard to whether and how they model latent appraisals of truth (or error in perceiving the truth), response biases, heterogeneous item difficulty, and cultural tendencies; furthermore, none of them have been extended yet to perform model-based clustering around consensuses or cultures. This is because they are generally concerned with discovering the ground truth, whereas the CCT model is interested in discovering the cultural truth(s). Hence, the prior models will aggregate across all cultures to estimate a ground truth while the CCT approach will seek to discover the consensus truth within each culture; and thus these models are formulated differently toward their intended functions. With that, the CCT approach of the CRM becomes further differentiated: on the basis of mathematical properties of our model, we derive techniques that suggest the appropriate number of consensus truths in the data, if one assumes the CRM to be applicable, and then develop posterior predictive checks to verify if the consensus structure of the data (number of cultures) is appropriately fit by the model.

The paper consists of five sections. After the Introduction, Section 2 specifies the new model. In Section 2.2, mathematical properties of the model are developed which are then used to form important model checks. The model is formulated within the hierarchical Bayesian framework in Section 2.3, posterior predictive checks are developed in Section 3.2, and appropriate application of the model is discussed in the other parts of Section 3. Section 4 demonstrates the model and its results on both simulated and real data sets. Section 5 provides the general discussion.

2. The CRM

2.1. Specification of the model

Assume that each of N informants provides a continuous answer within (0, 1), or within an allowable range that offers an appropriate linear transform to (0, 1), to each of M items, and let the responses be the realization of a random response profile matrix $\mathbf{X} = (X_{ik})_{N\times M}$, where

$$X_{ik} = x \quad \text{iff informant } i \text{ assigns value } x \text{ to item } k. \tag{1}$$

An item's value, or measure of truth, probability, or degree may be naturally interpretative or scalable to a value in (0, 1), and the informants have latent appraisals of these values with error. In such cases, the CRM links the (0, 1) locations of these values to the real line with the logit transform, where $\text{logit}(x) = \log(x/1 - x)$. Therefore, each item also has a consensus location in $(-\infty, \infty)$. The respondents have latent appraisals of these item values with a mean at the item's consensus location plus some error, which depends on the knowledge competency of the informant, and the difficulty of knowing the item. Then the observed responses are determined by the informant's response biases, which act on the latent appraisal value.

The CRM is formalized and further explained by the following axioms:

Axiom 1 (*Cultural Truths*)**.** There is a collection of $V \geq 1$ latent cultural truths, $\mathscr{T} = \{\mathbf{T}_1, \ldots, \mathbf{T}_v, \ldots, \mathbf{T}_V\}$, where $\mathbf{T}_v \in \prod_{k=1}^{M}(-\infty, \infty)$. Each informant i responds according to only one cultural truth (set of consensus locations), as $\mathbf{T}_{\Omega_i}$, where $\Omega_i \in \{1, \ldots, V\}$, and parameter $\mathbf{\Omega} = (\Omega_i)_{1\times N}$ denotes the cultural membership for each informant.

Axiom 2 (*Latent Appraisals*)**.** It is assumed that each informant draws a latent appraisal, Y_{ik}, of each $T_{\Omega_i k}$, in which $Y_{ik} = T_{\Omega_i k} + \epsilon_{ik}$. The ϵ_{ik} error variables are distributed normal with mean 0 and standard deviation σ_{ik}.

Axiom 3 (*Conditional Independence*)**.** The ϵ_{ik} are mutually stochastically independent, so that the joint distribution of the latent appraisals is given for all realizations (y_{ik}) by

$$h[(y_{ik}) \mid (T_{\Omega_i k}), (\sigma_{ik})] = \prod_{i=1}^{N}\prod_{k=1}^{M} f(y_{ik} \mid T_{\Omega_i k}, \sigma_{ik}) \tag{2}$$

where $f(y_{ik} \mid T_k, \sigma_{ik})$ is the normal distribution with mean $T_{\Omega_i k}$ and standard deviation σ_{ik}.

Axiom 4 (*Precision*)**.** There are knowledge competency parameters $\mathbf{E} = (E_i)_{1\times N}$, with $E_i > 0$, and item difficulty parameters specific to each cultural truth, $\mathscr{L} = \{\mathbf{\Lambda}_1, \ldots, \mathbf{\Lambda}_v, \ldots, \mathbf{\Lambda}_V\}$, where $\Lambda_v = (\lambda_{vk})_{1\times M}$, and each $\lambda_{vk} > 0$. An informant's standard appraisal error in the assessment of each $T_{\Omega_i k}$ is defined by standard deviation

$$\sigma_{ik} = E_i \lambda_{\Omega_i k}. \tag{3}$$

If all item difficulties are equal, then all λ_{vk} are set to 1.

[1] Greater success was arguably achieved using the Gaussian appraisal approach over the beta appraisal approach: the inverse logit values of the truths estimated in $(-\infty, \infty)$ spanned the full interval in (0, 1) more often than the values estimated in (0, 1) by the beta, informant knowledge precision of the truth was independent of item location with the Gaussian, and the use of polychoric correlations was available to help detect and adequately fit the consensus structure of the data.

[2] Data that is not in (0, 1) can be transformed to the interval with an appropriate linear transform, e.g. financial data from \$0 to \$100,000 would be divided by 100, 000. Secondly, indeed since the model uses the Gaussian approach, it may be used on data naturally within $(-\infty, \infty)$, though as discussed later, the logit transform of data in (0, 1) may be more appropriate for the approach developed here.