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From Uniform Expected Utility to Uniform Rank-Dependent Utility: An experimental study

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HIGHLIGHTS

- Uniform expected utility model for set rankings is experimentally tested.
- Averaging is violated by virtually all subjects.
- Restricted Independence holds for a subset of subjects.
- A new model is proposed, close to Rank-Dependent Utility.
- It fits all data published so far.

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ABSTRACT

This paper presents an experimental investigation of the Uniform Expected Utility (UEU) criterion, a model for ranking sets of uncertain outcomes. We verified whether the two behavioral axioms characterizing UEU, i.e., Averaging and Restricted Independence, are satisfied in a pairwise choice experiment with monetary gains. Our results show that neither of these axioms holds in general. Averaging in particular, appears to be violated on a large scale. On the basis of the current study and a previous one, we can conclude that none of the models for set ranking that have been axiomatically characterized so far is able to model observed choices between sets of possible outcomes in a satisfactory fashion. In this paper we therefore lay out the foundations for a new descriptive model for set ranking: the Uniform Rank-Dependent Utility (URDU) criterion.

1. Introduction

A far-famed approach to the modeling of uncertainty is the Bayesian one, which claims that, in the absence of objective probabilities, the decision maker should have her own subjective probabilities and these probabilities should guide her decisions. Another approach, not using probabilities, can be found in the literature about set rankings, surveyed by Barberà, Bossert, and Pattanaik (2004). In this domain, decisions are quite frugally described by nothing more than the sets of their possible outcomes. Comparing decisions hence reduces to comparing sets of possible outcomes.

The *Min and Max Induced Rankings* (MMIR) form a family of set rankings (Maximin, Maximax, Minmax, Maxmin, etc.) that require preferences over sets to be induced from comparison of the best and/or worst elements within those sets. The Minmax and Maxmin criteria (Arlegi, 2003; Bossert, Pattanaik, & Xu, 2000), for example, treat the best and worst elements in a lexicographical fashion. According to Minmax, comparison of the minima will be the primary criterion for ranking sets. In the case where the minima coincide, Minmax prescribes that the decision maker will proceed to comparing the maxima. An indifference will be stated if the minima as well as the maxima of both sets are identical. The Maxmin rule is the dual case in which the decision maker first considers the maxima in the sets to be compared, and when these are identical, she will go on to comparing the minima.

The Uniform Expected Utility (UEU) criterion, axiomatized by Gravel, Marchant, and Sen (2012), is another type of model for ranking sets of possible outcomes. The UEU criterion shows some similarities to the classical Expected Utility (EU) criterion in that it states that sets are ranked on the basis of the expected utility of their outcomes. In the absence of information about probabilities, however, it is assumed that the decision maker acts as if she considers all the possible outcomes of a decision as equally likely.

Most of the research in the field is pursued by theorists who are mainly concerned with the axiomatic characterizations of models

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for set ranking. From our point of view, however, it also seems interesting to investigate whether any of these models are capable of describing observed decision behavior.

So far, we know of only two studies that have adopted a descriptive approach in order to study set rankings: Vrijdags (2010) investigated whether rankings of sets of monetary consequences obey the transitivity axiom, and in a second paper, the MMIRs were examined empirically (Vrijdags, 2013).

In the current study, UEU will be the model under scrutiny. In specifically designed tests, reported in Vrijdags (2013), UEU appears to outperform the MMIRs in predicting the subjects' preferences. Yet, some observations were made that are hard to accommodate within the UEU framework. In Vrijdags (2013), subjects are asked to choose between sets of monetary consequences. When asked to choose between $A_1 = \{35, 4, 3\}$[1] and $B_1 = \{35, 3\}$, for example, 46% are estimated to prefer A_1. When confronted with the choice between $A_2 = \{20, 3, 2\}$ and $B_2 = \{20, 1\}$, as much as 78% of all participants opted for A_2. A similar choice is the one between $A_3 = \{20, 2, 1\}$ and $B_3 = \{20, 1\}$, where 38% stated a preference for set A_3. For a considerable share of subjects, it thus appears that they prefer one more outcome in the middle, instead of being constrained to a set with one high and one low outcome, even when the value of this middle outcome is very close to the minimum. This decision behavior might be explained by a positive attitude towards a diversification of uncertainty within the range of the minimum and the maximum of a set. Such choices – where a set with a considerably lower average is preferred – are hard to explain with UEU, unless one assumes an extremely risk averse utility function over the outcomes for all subjects choosing the three-elements sets with the lower arithmetic means over the outcomes. Although this seems rather implausible, it deserves empirical analysis. That is why we devote a large part of this paper to the empirical validation of the behavioral axioms, among those characterizing UEU.

The next section presents the model and the axioms that will be tested in the rest of the paper. Sections 3 and 4 will then present the empirical method and the results. In Section 6, we will discuss these results and propose a new promising model, close in spirit to the Rank-Dependent Utility model.

2. The uniform expected utility model

Let X be a non-empty universal set of outcomes, and let $\mathcal{X}$ denote the set of all non-empty, finite subsets of X. We assume the subjects have preferences over $\mathcal{X}$ that can be represented by a weak order (transitive[2] and complete binary relation) $\succsim$ over $\mathcal{X}$. The asymmetric part (strict preference) of $\succsim$ is denoted by $\succ$ while the symmetric part (indifference) is denoted by $\sim$.

We say that $\succsim$ is representable in the UEU model if and only if there exists a real-valued mapping u defined on X such that, for all $A, B \in \mathcal{X}$,

$$A \succsim B \iff \mathrm{UEU}(A) \geq \mathrm{UEU}(B),$$

where

$$\mathrm{UEU}(A) = \frac{1}{\#A} \sum_{a \in A} u(a).$$

Provided the relation $\succsim$ satisfies a richness condition, UEU has been characterized (Gravel et al., 2012) by means of two behavioral axioms (Averaging and Restricted Independence) and a technical condition (Archimedeanness). Sections 3–5 focus on these two conditions.

Averaging: for all disjoint sets A and $B \in \mathcal{X}$,

$$A \succsim B \Leftrightarrow A \succsim A \cup B \Leftrightarrow A \cup B \succsim B. \quad (1)$$

The Averaging axiom, first used by Fishburn (1972), ensures that enlarging a set A with a (disjoint) set of outcomes B that is not considered better than A is a worsening of the original set A. On the other hand, the axiom implies that enlarging B with a set A which is considered at least as attractive as B, constitutes an improvement of the original set B. The Averaging axiom is intended to capture an intuitive property satisfied by calculations of "average" in various settings.

Restricted independence: $\forall A, B, C \in \mathcal{X}$ with $\#A = \#B$ and $A \cap C = B \cap C = \emptyset$,

$$A \succsim B \Leftrightarrow A \cup C \succsim B \cup C. \quad (2)$$

The Restricted Independence axiom is a consistency condition which requires that the ranking of sets with equal cardinality is independent of any elements they may have in common. Hence, adding these common elements to or withdrawing them from the sets should not affect their ranking. A similar condition has been used in Nehring and Puppe (1996), although the latter is weaker because it constrains sets A and B to be singletons. Notice also the existence of a condition with a similar appearance in the literature on qualitative probability (e.g., Krantz, Luce, Suppes, & Tversky, 1971, p. 204); there, A and B are events rather than sets of outcomes, and they do not necessarily have the same cardinality; it is used to derive an additive representation of a probability.

3. Method

3.1. Choice stimuli

Our goal is to determine to what extent UEU applies through an investigation of its characterizing behavioral axioms: Averaging and Restricted Independence.

If the subjects' choices do not obey Averaging and/or Restricted Independence, we know that they do not decide according to the UEU model. When devising experiments for axiom tests, one usually tries to "challenge" the axiom under consideration by selecting choice objects or stimuli that are expected to be capable of refuting an axiom if it does not hold in general. Previous research has led us to believe that the choices of people who do not appear to follow UEU might be guided by an inclination towards a diversification of the possible outcomes within the range bounded by the minimum and the maximum of the set. Consequently, where possible, we used this assumption when devising the choice stimuli for this study.

Table 1 shows the pairs of sets that were used to investigate the descriptive validity of Averaging and Restricted Independence. These pairs of sets are administered to the subjects in a forced choice experiment. The instructions of the experiment explain that the numerical set elements represent monetary amounts in €. Each set can be conceived of as a lottery in the form of a container holding one hundred tickets. On each of these tickets, one of the amounts in the set is printed. However, the frequency distribution of the different tickets in the container is unknown. For example, a set {30, 23} can be thought of as a container holding an unknown number of tickets with "30" printed on them as well as an unknown number of tickets with "23" printed on them, both of which sum to one hundred. In order to play the lottery, one ticket would be drawn at random from the container, and the amount

1 Set A_1 can be thought of as an occasion to win either €35, €4, or €3 with unknown probabilities. A more detailed account of how the subjects are instructed to conceive of the sets they are presented with can be found in the Method section.

2 We know from Vrijdags (2010) that transitivity is a reasonable hypothesis in this context.