Journal of Epidemiology and Global Health (2014) 4, 107–114

http://www.elsevier.com/locate/jegh

Effects of habitat characteristics on the growth of carrier population leading to increased spread of typhoid fever: A model

J.B. Shukla [a], Ashish Goyal [b,*], Shikha Singh [c], Peeyush Chandra [c]

[a] Bhabha Group of Institutions, Kanpur (Rural) 209204, India
[b] The University of New South Wales, Sydney, NSW 2052, Australia
[c] Department of Mathematics and Statistics, IIT Kanpur, Kanpur 208016, India

Received 25 July 2013; received in revised form 10 September 2013; accepted 11 October 2013
Available online 5 December 2013

KEYWORDS
Habitat characteristics;
Carriers;
Immigration;
Logistic model;
Stability

Abstract In this paper, a non-linear model is proposed and analyzed to study the effects of habitat characteristics favoring logistically growing carrier population leading to increased spread of typhoid fever. It is assumed that the cumulative density of habitat characteristics and the density of carrier population are governed by logistic models; the growth rate of the former increases as the density of human population increases. The model is analyzed by stability theory of differential equations and computer simulation. The analysis shows that as the density of the infective carrier population increases due to habitat characteristics, the spread of typhoid fever increases in comparison with the case without such factors.

1. Introduction

Typhoid fever is a bacterial disease caused by Salmonella typhi. It is considered as a burden with the highest incidence rates of the disease in Africa and Asia [1]. It is transmitted through the ingestion of food or drink contaminated with bacteria which may be transported by carriers such as flies from the feces or urine of infected people. Typhoid fever spreads in the population because of two factors: (i) carriers such as flies, which transport bacteria of disease from excreta of those infected to susceptible individuals; and (ii) direct contact between those infected and susceptible individuals [2]. The changes in the cumulative density of habitat characteristics, such as plant and vegetation in residential areas, open drainage, garbage dumps, water storage tanks, ponds, etc., provide a very conducive environment for breeding, growth and survival of carriers such as flies leading to the increased spread of typhoid fever [3]. It is noted that the cumulative density of these habitat characteristics may increase due to human population density-related factors such as lack of proper sanitation, water contamination, etc.

---

* Corresponding author. Tel.: +61 0420770287.
E-mail address: ashish.goyal@student.unsw.edu.au (A. Goyal).

http://dx.doi.org/10.1016/j.jegh.2013.10.005

It is pointed out here that the study of habitat characteristics representing ecological and environmental factors such as the above-mentioned and their effects on the growth of the carrier population is very complex [4—13]. In this paper, a simple non-linear model is proposed to study this aspect on the spread of typhoid fever.

The modeling and analysis of various infectious diseases have been conducted by many researchers in the past [14—23]. Although some research has been conducted on the carrier-dependent infectious diseases [21], the effect of the cumulative density of habitat characteristics on the carrier population has not been considered. It is noted that very little attention has been paid to the study of typhoid fever by considering effects of the carrier population, the density of which increases due to natural as well as human population density-related habitat characteristics. Therefore, in this paper, a SIR (susceptible-infected-recovered) model with constant immigration for carrier-dependent infectious disease is proposed and analyzed by considering explicitly the effects of habitat characteristics. To be specific in the modeling process, the cumulative density of habitat characteristics (such as biomass of leaves in bushes and plants, etc.) is considered to be governed by a logistic model, the growth rate of which increases as the density of the human population increases. It is assumed further that the density of carriers is also governed by a logistic model, the growth rate of which increases as the cumulative density of habitat characteristics increases. Although there are multiple other factors besides habitat characteristics that are associated with the spread of typhoid fever, such as seasonality, water contamination, sanitary practices, etc., the model focuses solely on the role of habitat characteristics on the spread of typhoid fever.

2. SIRS model with ecological effects

Let time total human population density $N(t)$ be divided into three sub-classes: the susceptible density $X(t)$, the infected density $Y(t)$ and the recovered density $R(t)$, thus $N = X + Y + R$. Let $B(t)$ be the cumulative density of habitat characteristics favorable to the growth of the carrier population. It is assumed that this density $B(t)$ is governed by a logistic model and growth rate of which increases as the human population density increases. Further, let $C(t)$ be the carrier population density also governed by a logistic model whose growth rate is favored by habitat characteristics. Also, $C_i(t)$ is the fraction of the carrier population density C which carries infective agents to the susceptible individuals.

Keeping in mind the above factors, and by assuming simple mass action interaction, a SIR model is proposed as follows:

$$\begin{aligned}
dX/dt &= A - \beta XY - \lambda XC_i + v_1R - dX,\\
dY/dt &= \beta XY + \lambda XC_i - (v + \alpha + d)Y\\
dR/dt &= vY - (v_1 + d)R,\\
dC_i/dt &= s_1C - s_{10}C_i,\\
dC/dt &= s_0(C - C^2/L) - s_1C + s_2BC,\\
dB/dt &= r_0B - r_0B^2/K - r_1B + r_2BN.
\end{aligned} \tag{2.1}$$

In model (2.1), A is the constant immigration rate of the human population; d is the natural death rate constant; β and λ are the transmission coefficients due to the infected human population [2] and the infected carrier populations respectively; v_1 is the fraction of R becoming susceptible again; α is the disease-related death rate constant; v is the recovery rate constant; s_1 is the rate at which carriers become infected carriers; and s_{10} is the death rate coefficient of infected carriers due to natural factors as well as by control measures. Also, s_2 is the growth rate of carriers because of the conductive habitat characteristics. Further, r_0 is the natural growth rate coefficient of $B(t)$; r_1 is the natural depletion/control rate of $B(t)$; r_2 is the growth rate coefficient of $B(t)$ due to human population density-related factors; and K is the carrying capacity of $B(t)$, which is assumed to be a constant. Similarly, s_0 is the growth rate, and L is the carrying capacity of the carrier population.

3. Equilibrium analysis

For analysis of the model (2.1), the following reduced system is considered (using $X + Y + R = N$):

$$\begin{aligned}
dY/dt &= \beta(N - Y - R)Y + \lambda(N - Y - R)C_{i(} - (v + \alpha + d)Y,\\
dR/dt &= vY - (v_1 + d)R,\\
dN/dt &= A - dN - \alpha Y,\\
dC_i/dt &= s_1C - s_{10}C_i,\\
dC/dt &= s_0(C - C^2/L) - s_1C + s_2BC,\\
dB/dt &= r_0B - r_0B^2/K - r_1B + r_2BN.
\end{aligned} \tag{3.1}$$

To analyze the model (3.1), the following lemma is needed which is stated without proof. This lemma establishes a region of attraction for the system.

Lemma 3.1. *The set*

$$\Omega = \{(Y, N, C, B) : A/(\alpha + d) \leqslant Y + R \leqslant N \leqslant A/d, 0 \leqslant C_i \leqslant s_1C_m/s_{10}, 0 \leqslant C \leqslant C_m, 0 \leqslant B \leqslant B_m\}$$