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Nuts and Bolts in Clinical Research

A look at the various ratios in medicine – risk ratio, odds ratio and likelihood ratio

Sourabh Dutta*

Division of Neonatology, Department of Pediatrics, PGIMER, Chandigarh, India

ARTICLE INFO

Article history:
Received 30 May 2014
Accepted 6 June 2014
Available online 28 July 2014

Keywords:
Ratio
Statistics
Proportion analysis

ABSTRACT

This article focuses on 3 kinds of ratios commonly used in biostatistics and research: risk ratio, odds ratio and likelihood ratio. It clarifies the meaning of risk and odds and then delves into the respective ratios.

1. Ratios

There are 3 kinds of ratios very commonly used in biostatistics and research: risk ratio, odds ratio and likelihood ratio. These ratios are deceptively simple, but they incorporate many nuances that are inapparent at first glance.

In general terms, a "ratio" is a number divided by another number where the units of both the numbers is the same. Since the units cancel off, the ratio by itself is unitless.

1.1. Risk

Before we try to understand the meaning of risk ratio, let us try to understand the meaning of "risk". Risk is simply the probability of the occurrence of a binary event. By convention, the word "risk" is used when the event in question is undesirable. For example, one talks of the risk of dying but not the risk of surviving; and the risk of failing an exam but not the risk of passing an exam! The denominator for risk is the study population in which one is looking for the event of interest. In simple terms, the study population is a group of subjects. Some subjects in the study population have the event of interest and some don't.

Imagine we have a population of 100 subjects of whom the "red" ones have the event of interest (say disease) and the "green" ones do not (Fig. 1A). If we sort out the "red" subjects and the "green" subjects, we find that 40 of the 100 subjects have the event of interest (Fig. 1B). Thus the probability of having the event of interest – or in other words the risk of the event – is 40/100; which can also be written as 40% or 0.40. If we convert this information into a pie diagram, we get a "red" slice that occupies 40% of the full size of the pie (Fig. 1C).

Risk is the slice of the pie divided by the whole pie. Risk is a ratio where the numerator is a fraction of the denominator. Therefore, the risk of an event can never be greater than 1. The risk of an event can extend only from 0 to 1.

* Tel.: +91 172 2755313 (office); fax: +91 172 2744401.
E-mail address: sourabhdutta1@gmail.com.
http://dx.doi.org/10.1016/j.injr.2014.06.002

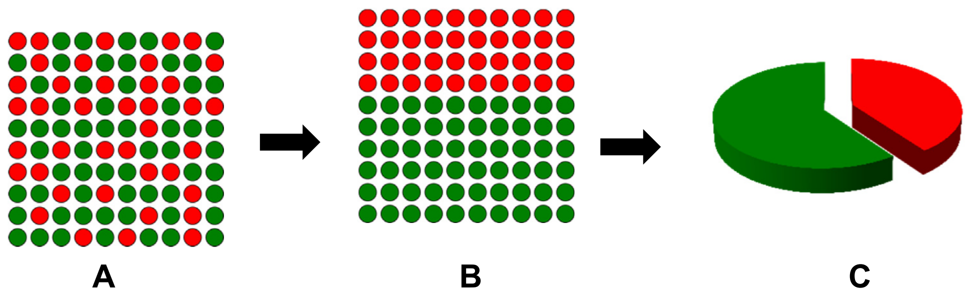

Fig. 1 – Odds and risks – understanding them graphically.

1.2. *Odds*

Odds is closely related to risk and it is possible to derive one from the other. In the above example, if we calculate a ratio of subjects who had the event of interest divided by subjects who did not have the event of interest, it would be called "Odds". In other words, we would be dividing the number of "red" subjects by the number of "green" subjects. We would be comparing the red slice of the pie with the remainder of the whole pie (i.e. the green slice).

Notice that the difference of odds with risk lies in the denominator. The risk has the whole study population (or the whole pie, if you may) as the denominator; whereas, odds has the remainder of the study population (who did not have the event of interest) as the denominator. For a given study population, the odds will always be greater than the risk of an event, because the denominator in case of odds is always less than the denominator in case of risk, whereas the numerator is the same for both. In the above example, odds would be 40:60, or 0.66.

Nothing prevents the red slice from being larger than the green slice. In another study population, there could very well have been 55 diseased subjects and 45 non-diseased subjects. The odds – in that case – would be 1.22. Since the numerator is not a fraction of the denominator, the odds of an event may be more than 1. It may extend from 0 to infinity.

If very few subjects have the event of interest, the odds start approximating the risk; because in that case the size of the green slice is almost equal to the size of the whole pie. Since the denominator of odds becomes almost the same as the denominator for risk, odds and risk become virtually the same.

From the diagram above, is that it is easy to understand that odds and risk can be derived from each other (where the risks and odds are expressed as decimals and not percentages):

$$\text{Odds} = \text{Risk}/(1 - \text{Risk})$$

Transforming the question, it turns out that:

$$\text{Risk} = \text{Odds}/(1 + \text{Odds})$$

2. Risk ratio

Until now we have been discussing the description of a binary event in a single group of subjects. Very often, in medical research we are interested in comparing 2 groups of subjects. For instance, in a randomised controlled trial we compare the group that received the intervention versus the group that received placebo. In a case-control study, we compare cases who have the disease of interest versus controls who do not have the disease of interest. In cohort studies, we compare subjects who are exposed to risk factor versus subjects who are not exposed to a risk factor.

In each group of subjects, we can calculate the risk of having the event. The ratio of the risks in the 2 groups is called risk ratio. Risk ratio is also called relative risk. The 2 terms can be used interchangeably. As we have seen earlier, risk by itself is a ratio. Therefore, the risk ratio is a ratio of 2 ratios.

By convention, in a randomised controlled trial, the numerator is the risk in the intervention arm and the denominator is the risk in the standard treatment arm. In a cohort study, the numerator is the risk in the exposed arm and the denominator is the risk in the unexposed arm.

In Fig. 2, two groups are being compared. The risk in group 1 is 40/100, i.e. 0.4. The risk in group 2 is 80/100, i.e. 0.8. The risk ratio is 0.4/0.8 = 0.5. In plain English this means that the risk of the event (say disease) in group 1 is 50% of what it is in group 2. In each of the groups, the risk is a fraction. However, in the risk ratio, the numerator is not a fraction of the denominator. Therefore, risk ratios can range anywhere from 0 to infinity.

If the risk ratio is equal to 1, it means the risk of having the event is the same in both groups. In a randomised controlled trial a risk ratio of 1 means that the risk of having the outcome in the intervention arm is no different from the risk of having the outcome in the placebo arm. In a cohort study, a risk ratio of 1 means that the risk of having the outcome of interest is no different between the exposed and the unexposed groups.

In a randomised controlled trial, a risk ratio of less than 1 means that the intervention is better than the placebo in the study population. If the entire 95% confidence interval of the risk ratio is on the lower side of 1, we can infer that the

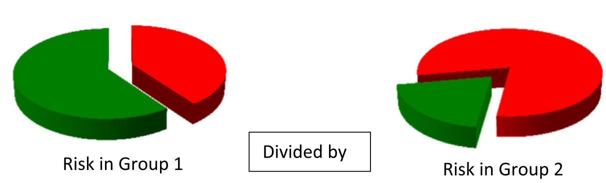

Fig. 2 – A graphical understanding of risk ratio.