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Using CAS to solve a mathematics task: A deconstruction

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ABSTRACT

I investigate how and whether a heterogeneous group of first-year university mathematics students in South Africa harness the potential power of a computer algebra system (CAS) when doing a specific mathematics task. In order to do this, I develop a framework for deconstructing a mathematics task requiring the use of CAS, into its primary components. This framework is based on the semiotic notion of diagrammatic reasoning whereby reasoning consists of construction of signs, transformation of signs, and observation and interpretation of signs. I use the framework to distinguish between the activities of students who were computer literate on entry to university and those who were not computer literate. The analysis suggests that formerly non-computer literate students are no worse than computer literate students in using CAS to construct various representations of signs, but that they are less able to interpret these signs. I propose that, in the South African context, this is largely due to inequities in prior mathematical education, rather than a lack of computer literacy per se.

1. Background

In the last three decades, various mathematics educators have advocated the use of technologies such as graphic calculators, dynamic geometry systems or computer algebra systems (CAS)[1] as tools in the learning of mathematics. Many mathematics educators have argued that the ability of the user to use the technology to move between different representations of mathematical objects promotes conceptual growth (e.g., Heid & Blume, 2008; Tall, 2000). Dörfler (1993) has suggested that the separation of the execution of mathematical tasks (performed by the computer) from the planning of mathematical tasks (carried out by the learner) could result in an increased focus on conceptual planning and problem-solving. Related to this, it is argued that the use of computer algebra systems may eliminate the need for cumbersome symbol manipulations so freeing students to concentrate on the formulation of solutions (Palmiter, 1991).

Although it is generally agreed that various forms of technology may enhance students' understanding of mathematics (Zbiek & Hollebrands, 2008) and may promote deeper understanding of advanced mathematical concepts (NCTM, 2000), it is also recognized that "the availability of technology does not ... guarantee enhanced learning" (Heid & Blume, 2008, p. 424). Heid and Blume argue that the sorts of activities with the technology (tasks) and the opportunity for reflection are of fundamental importance.

A further motivation for the use of CAS in a university mathematics course, is its extensive use by mathematicians in the world of work. For example, CAS is used in financial institutions, engineering companies and commercial enterprises and all mathematics graduates of the 21st century need to be competent in its use.

In the South African context, the use of technology in the learning of mathematics at university level is complicated by equity issues: some students have grown up with a computer in their bedroom; other students have never seen, let alone used, a computer. Concerns about equity makes it necessary to consider the extent to which students who are not computer literate on entry to university are disadvantaged by the use of computers in a first-year mathematics course and, if so, how. In this regard, Dunham and Hennessy (2008, p. 401) argue that "researchers have a role to play in reducing technology inequities by helping the education community to better understand the source of inequities and to devise and affirm appropriate instructional strategies and public policies for addressing these gaps". This paper will hopefully contribute to an understanding of the nature of inequities (although developing adequate instructional strategies and policy is beyond the scope of the paper).

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[1] CAS is software which transforms the computer into a powerful calculator which may be used to generate graphs or to manipulate symbols (as well as numbers) in a mathematical way. It also has many inbuilt mathematical functions and users may define their own mathematical operations.

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1.1. Focus

On the theoretical side, I develop a framework within which to deconstruct a CAS-based task into its major components. Using this framework, I investigate whether and how a heterogeneous group of first-year university mathematics students in South Africa are able or not to harness the potential power of a computer algebra system (CAS) when doing a CAS-based mathematical task[2] I pay particular attention to the group of students who entered the university without any prior experience of computers.

2. A semiotic framework

C.S. Peirce (1839–1914), one of the founding fathers of semiotics, argued that signs are not only a means of signifying or referring to an object; rather they are "means of thought, of understanding, of reasoning and of learning" (Hoffmann, 2005, p. 45).

The use of a semiotic framework when looking at mathematical activities (be it with a computer or not) has become more and more widespread in mathematics education (for example, Dörfler, 2006; Hoffmann, 2005; Radford, 2000). The appeal of such a framework lies in its central tenet: signs (such as words, symbols, graphs, diagrams) and thinking co-exist. Neither can exist without the other and both evolve with each other. In this vein, Hegedus and Moreno-Armella (2008) give an illuminating example of how thinking and using signs (writing) are inextricably linked. They quote a conversation taken verbatim from Gleik's (1992) biography of the famous scientist, Richard Feynmann and the historian, Charles Weiner. In this conversation, Weiner mentions that Feymann's notes are "a record of the day-to-day work". Feynmann retorts: "I actually did the work on paper". Weiner then suggests: "The work was done in your head, but the record of it is still here" to which Feynmann counters : "No, its not a record, not really. It's working. You have to work in paper, and this is the paper". In this brief excerpt Feynmann implicitly endorses a semiotic perspective: his thinking and his writing mathematics are inextricably interwoven and mutually constitutive.

Peirce argued that all deductive reasoning, such as that used in mathematical thinking could be explained in terms of three major components, all relating to the mediating use of signs. These components are: *constructing* a representation (a set of signs), experimenting with these representations through manipulations or in the imagination (that is, *transforming* the signs) and *observing* the results. He regarded the set of signs as a diagram and so called this form of mathematical reasoning, 'diagrammatic reasoning'. "All deductive reasoning ... involves an element of observation: namely, deduction consists in *constructing* an icon or diagram the relation of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of *experimenting* upon this image in the imagination, and of *observing* the result so as to discover unnoticed and hidden relations among the parts" (my italics, Peirce, Collected Papers 3.363 – cited in Dörfler, 2006, p. 102).

For example: a common pencil and paper way of solving the problem "Find the roots of $\frac{2x^2-x-6}{x-2}$", is the following: the mathematics student *constructs* the sign: $\frac{2x^2-x-6}{x-2}=0$; she *experiments* with the signs through *transformations* of the written expression. For example, she may write $\frac{(x-2)(2x+3)}{(x-2)}=0$. Depending on the student and her prior knowledge, the student *observes* that $x \neq 2$. She may then further *transform* the signs in her imagination or on paper by writing: $2x + 3 = 0$. Finally the student manipulates (*transforms*) the symbols to get $x = -3/2$ which she *interprets* to mean that the solution to the equation is $x = -3/2$. The point is: it is the student's construction, manipulation of signs (written or in the imagination) according to the rules of mathematics, and observations again according to the rules of mathematics, which leads her to the correct solution.

I suggest that Peirce's categorization of mathematical reasoning (as diagrammatic reasoning) is particularly useful for isolating key components of mathematical tasks. In this paper I use diagrammatic reasoning as the basis for a framework within which I deconstruct a CAS-based task into its major components. I then examine how a group of first-year university mathematics students engages with a specific CAS-based task in terms of these components.

2.1. Diagrammatic thinking applied to CAS

Broadly speaking most CAS-based tasks involve the *construction* of one or more CAS-based signs for the mathematical object or operation of interest, and *observation* and *interpretation* of the CAS output. This interpretation may result in the *transformation* of the CAS output (the CAS sign) into further signs. I elaborate on these activities here.

2.1.1. Construction of sign

A student engaging in CAS-based activities will, at some point, need to construct a suitable mathematical sign on CAS. This may be a representation of the mathematical object (for example, a graph or the definition of a function) and/or it may be an operation (for example, Solve [*f*[*x*] == *g*[*x*], *x*]).[3] In order to construct the CAS-based sign, the student needs to be familiar with the appropriate CAS syntax and she needs basic technical skills (e.g., keyboard skills) for using the CAS (see Pierce & Stacey, 2004). She also may need specific mathematical skills or knowledge. For example, if she wishes to plot a sketch of arcsine *x*, she needs to know the appropriate syntax, i.e. Plot[ArcSin[*x*],{*x*, min domain, max domain}], and she needs to know that the domain of ArcSin *x* is $[-1, 1]$. She also needs to know that she must press Shift and Enter simultaneously in order for the computer to execute her command.

The construction of graphs in the CAS environment presents its own challenges (Tall, Smith, & Piez, 2008); in particular, the choice of an appropriate domain may be problematic (Artigue, 2002; Goldenberg, 1988). In the pencil and paper environment, drawing an unfamiliar function by hand requires a prior analysis of the function. That is, the student first needs to analyse the function by finding key features such as turning points, points of inflection, asymptotes, intercepts, of the graph, if they exist. From this the student deduces an appropriate domain. With graphics software, accurate prior analytic work is not necessary. Rather students use greater or lesser knowledge about the functions to presume a roughly useful domain and then use trial and error to get what they deem an appropriate domain. The problem is that key features (e.g., turning points) of the graph are often missed in this experiential environment.

[2] A CAS-based task is a mathematics task which requires the use of CAS and possibly pencil and paper.

[3] All CAS examples that I offer are generated using *Mathematica*.