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Learning from comparison in algebra

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ABSTRACT

Mastery of algebra is an important yet difficult milestone for students, suggesting the need for more effective teaching strategies in the algebra classroom. Learning by comparing worked-out examples of algebra problems may be one such strategy. Comparison is a powerful learning tool from cognitive science that has shown promising results in prior small-scale studies in mathematics classrooms. This study reports on a yearlong randomized controlled trial testing the effect of an Algebra I supplemental comparison curriculum on students' mathematical knowledge. 141 Algebra I teachers were randomly assigned to either implement the comparison curriculum as a supplement to their regular curriculum or to be a 'business as usual' control. Use of the supplemental curriculum was much less frequent than requested for many teachers, and there was no main effect of condition on student achievement. However, greater use of the supplemental curriculum was associated with greater procedural student knowledge. These findings suggest a role for comparison in the algebra classroom but also the challenges of supporting teacher integration of new materials into the curriculum.

1. Introduction

Mastery of algebra is an important milestone for students. Algebra serves as a gatekeeper for citizenship (Education Commission of the States, 1998) and also provides students with the ability to harness new technologies and take advantage of the job opportunities resulting from them (Moses & Cobb, 2001). Furthermore, success in algebra is necessary for access to higher mathematics and is correlated with positive life outcomes such as college graduation (Adelman, 2006; National Mathematics Advisory Panel, 2008).

Unfortunately, many students struggle with algebra. In particular, students often find the transition from arithmetic to algebra difficult (e.g., National Research Council, 2001). Algebra is the first time in mathematics where students engage in prolonged abstraction and symbolization (Kieran, 1992), for example, by frequently working with symbols that have an abstract meaning, such as variables (e.g., '*x*'). National and international assessments have drawn attention to pervasive student difficulties in algebra (e.g., Beaton et al., 1996; Blume & Heckman, 1997; Lindquist, 1989; Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999). For example, eighth-grade NAEP data show that students continue to struggle on very straightforward algebra problems: Only 59% of 8th graders were able to find an equation that is equivalent to $n + 18 = 23$, and only 31% of 8th graders were able to find an equation of a line that passes through a given point and with a negative slope (National Assessment of Educational Progress, 2011).

Improving students' mastery of algebra requires finding effective strategies for teaching and learning algebraic topics. To this end, we report the results of a yearlong intervention based on the application of a promising approach from cognitive science to the teaching and learning of mathematics – namely, contrasting and comparing examples. In particular, we tested (1) whether the use of a supplemental comparison curriculum increased students' knowledge in algebra and (2) if greater use of the curriculum materials was associated with greater student knowledge in algebra. In the following section, we motivate the present study by discussing prior research on learning through comparison, both in the laboratory and the classroom.

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2. Background on comparison

Comparison is a powerful tool that has been shown to improve learning in a variety of domains. In both laboratory studies (e.g., Gentner, Loewenstein, & Thompson, 2003; Kurtz, Miao, & Gentner, 2001; Namy & Gentner, 2002) and small-scale classroom studies (e.g., Rittle-Johnson & Star, 2007), having learners compare and contrast worked examples has been shown to reliably lead to gains in students' knowledge (see Rittle-Johnson & Star, 2011, for a review). For example, infants can learn to distinguish between cats and dogs by comparing a picture of a cat and a picture of a dog, side-by-side (Oakes & Ribar, 2005). By comparing the two pictures (or several sets of dog-cat pairs), infants may better understand important features that distinguish cats from dogs and thus learn to distinguish between the animals more easily than if the infants learned about each animal separately. Generally, comparing side-by-side examples can help individuals understand important features of a problem, which in turn may aid with novel problem solving (Gentner et al., 2003) as well as future learning (Schwartz & Bransford, 1998).

In addition, comparison is integral to best practices in mathematics education. Having students share solution procedures for a particular problem and then discuss the similarities and differences in the different procedures lies at the core of reform pedagogy in many countries throughout the world (e.g., Australian Education Ministers, 2006; Brophy, 1999; Kultusministerkonferenz, 2004; National Council of Teachers of Mathematics, 2000; Singapore Ministry of Education, 2006; Treffers, 1991), including the Common Core Standards (Common Core State Standards Initiative, 2010) in mathematics in the US. At all grade levels, teachers are encouraged to create an environment where students can engage in thinking and communicating deeply about mathematics—in discussing, collaborating, justifying, conjecturing, experimenting, and responding to the ideas of their peers (National Council of Teachers of Mathematics, 2000, p. 18). The development of mathematical understanding is believed to be enhanced by classroom discussions, where students share procedures and evaluate the procedures of others (Lampert, 1990; Silver, Ghousseini, Gosen, Charalambous, & Strawhum, 2005), including informal and non-standard algorithms (Carroll, 2000; Mack, 1990). Expert teachers in the U.S. as well as teachers from high-performing countries have students compare different ways to solve the same math problem (e.g., Ball, 1993; Lampert, 1990; Richland, Zur, & Holyoak, 2007). A recent Practice Guide from the US Department of Education (Woodward et al., 2012) identified comparison as one of five recommendations for improving mathematical problem solving in the middle grades.

This practice guide recommendation is largely based on a number of small-scale experimental classroom studies documenting the benefits of comparison to students' learning of mathematics (Guo & Pang, 2011; Rittle-Johnson & Star, 2007; Rittle-Johnson & Star, 2009; Rittle-Johnson, Star, & Durkin, 2009; Rittle-Johnson, Star, & Durkin, 2012; Star & Rittle-Johnson, 2008, 2009). Each of these studies include two common features of experimental research on the benefits of comparison – the use of worked examples and prompts for explanation – both of which have been shown to improve learning (e.g., Atkinson, Derry, Renkl, & Wortham, 2000; Pashler et al., 2007). A general finding from prior small-scale studies on comparison is that students who are shown two worked examples side-by-side and given the opportunity to compare and discuss similarities and differences between problems, solutions, and strategies significantly outperform control students on a variety of outcome measures.

For example, in a one-week-long experimental study, Rittle-Johnson and Star (2007) randomly assigned 70 7th grade student pairs to learn multistep linear equation solving by either comparing two worked-out examples presented side-by-side, or by studying isomorphic worked-out examples sequentially. Using a pretest–intervention–posttest design, the authors found that students in the comparison condition demonstrated greater procedural and flexibility knowledge than students in the sequential condition. In a similar one-week-long study involving 157 5th and 6th grade students, Star and Rittle-Johnson (2009) extended the benefit of comparison to a second domain of mathematics: computational estimation. Using the same research design, the authors showed that students in the comparison condition demonstrated greater flexibility knowledge than students in the sequential condition. Comparison has also been shown to improve fourth graders' learning about the altitude of a triangle (Guo & Pang, 2011).

Thus, there is emerging evidence from laboratory studies in cognitive science and from short-duration, researcher-led classroom studies on the benefits of comparison; the above-mentioned Practice Guide noted that there is moderate evidence in support of this practice (Woodward et al., 2012). Yet research is lacking on the potential for comparison to improve long-term learning in classrooms. In a study examining feasibility of classroom implementation, Newton, Star, and Lynch (2010) demonstrated that, through comparison of worked examples, struggling 9th, 10th, and 11th grade students were able to learn and appreciate multiple strategies for solving problems during a three-week, researcher-led algebra course. However, no prior study has examined the impact of comparison on students' learning of mathematics in authentic (e.g., teacher-enacted, full-year-long) classroom environments. The present study seeks to fill this gap, by evaluating the impact of teachers' use of a supplemental Algebra I comparison curriculum. We adapted and expanded intervention materials from prior studies on comparison (e.g., Rittle-Johnson & Star, 2007) to create a full-year-long supplemental curriculum; we provided an intensive professional development to teachers on how to implement the curriculum; and we evaluated fidelity of teachers' implementation of the curriculum as well as their students' learning gains.

2.1. Supplemental comparison curriculum

Analyses of large-scale efforts to reform mathematics instruction suggests that when an innovation closely resembles current practices and is easy to implement, teachers are more likely to adopt the innovation (Cohen & Hill, 2001). As a result, we chose to supplement rather than replace teachers' current Algebra I curriculum with easy-to-implement materials designed to infuse comparison into teachers' regular practice. Specifically, a team of mathematics education experts, including researchers, mathematicians, and Algebra I teachers, developed the materials by going through a typical Algebra I course syllabus, identifying common student difficulties and misconceptions, and then creating materials to attempt to address them. Semi-structured interviews with a small group of teachers confirmed that comparison was indeed a reasonable adaptation of their current practice (e.g., many of the teachers introduced multiple strategies for at least some problem types, but they did not explicitly compare the strategies). These teachers piloted the supplemental comparison curriculum for a full-year prior to the present study.

At the core of the supplemental Algebra I curriculum were 141 worked example pairs (WEPs). Each WEP showed the mathematical work and dialogue of two hypothetical students, Alex and Morgan, as they attempted to solve one or more algebra problems. The curriculum contained four types of WEPs, with the types varying in what is being compared and the instructional goal of the comparison (see Fig. 1). Two of the WEP types were very similar in content and format to the intervention materials used in prior work. First,