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Pathways to arithmetic: The role of visual-spatial and language skills in written arithmetic, arithmetic word problems, and nonsymbolic arithmetic

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ABSTRACT

This study set out to develop and test a pathway model of the relations between general cognitive skills, specifically visual-spatial and spoken and written language skills, and competence in three forms of arithmetic that vary in modes of number representation. A total of 88 Chinese 4-year-olds participated and were tested first in kindergarten second grade (K2) and then in kindergarten third grade (K3). Language skills, including phonological, morphological, and visual-orthographic skills, and visual-spatial skills were measured at K2, and arithmetic outcomes, including written arithmetic, word problems, and nonsymbolic arithmetic, at K3. The results generally supported our model. Specifically, visual-spatial skills contributed to the prediction of all three types of arithmetic outcomes. Morphological skills predicted word problems, whereas phonological skills predicted written arithmetic. Finally, visual-orthographic skills contributed to both written and nonsymbolic arithmetic. These findings underscore the importance of delineating the specificity of cognitive processes in learning diverse forms of arithmetic.

1. Introduction

Numbers can be represented in at least three modes: Arabic digits (e.g., 6), words (e.g., six), and nonsymbolic quantities (e.g., six balls). Correspondingly, written arithmetic (e.g., $4 + 2 = ?$), arithmetic word problems (e.g., "Ivy has four balls. Her sister gives her two more balls. How many balls does Ivy have now?"), and nonsymbolic arithmetic (e.g., the operation of adding two real balls to a collection of four) are often regarded as the three central forms of arithmetic (Levine, Jordan, & Huttenlocher, 1992). Achieving competence in these forms at an early age provides the basis for learning more advanced mathematics later (e.g., Geary, Hoard, Nugent, & Bailey, 2013) and even for success in the labor market well into adulthood (Every Child a Chance Trust, 2009). However, our knowledge of the sources that contribute to early arithmetic learning is limited. Only a few models have been proposed, and they suggest that general cognitive abilities, such as visual-spatial and language skills, provide the building blocks for arithmetic learning (Krajewski & Schneider, 2009; LeFevre et al., 2010). Most of these models do not consider the *multiple* forms of arithmetic. Furthermore, most prior work largely ignores the role of written language in learning arithmetic. To fill these gaps, this study reports on the development and testing of a pathway model of the relations between visual-spatial and language (i.e., spoken versus written) skills and young children's competencies in three forms of arithmetic varying in modes of representation, including written arithmetic, word problems, and nonsymbolic arithmetic.

1.1. Modes of representation and arithmetic development

Representations play an important role in the learning and teaching of mathematics (Bruner, 1966; Goldin, 2008; Kaput, 1987). The flexible use of various representations is even regarded as one of the goals of mathematics education (National Council of Teachers of Mathematics, 2000). In an analysis of mathematical problem solving, Lesh (1981; Lesh, Landau, & Hamilton, 1983) suggests that modes of representation are diverse and include, but are not limited to, written symbols, spoken symbols, and manipulative models.

Children develop their understanding of arithmetic concepts by using different modes of representation and connections among them (Lesh et al., 1983). Theories of arithmetic development posit that a fundamental understanding of addition and subtraction evolves from children's early experience with concrete objects

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(Gelman & Gallistel, 1978; Ginsburg, 1977; Resnick, 1992). For example, by playing a collection of marbles, young children recognize that adding one marble to a collection of three makes a collection of four and that taking away one marble makes a collection of two. Later on, children learn about numerical and arithmetic symbols in school and connect them with the objects they represent (Ginsburg, 1977). That is, in order to perform the addition or subtraction of two numbers, the spoken or written number symbols must be aligned with the number of objects in a set, and the arithmetic operator must be connected to the operation of adding one set to the other or taking away one set. Moreover, children can also use one symbolic mode of representation to expand and understand another (Leinhardt, Zaslavsky, & Stein, 1990). For instance, children may learn written numerals by connecting them with spoken number words. Eventually children grasp the ability to perform translations among and transformations within various modes of representation (Lesh et al., 1983). Such a flexible integration of the different modes of representation is considered as a fundamental process leading to arithmetic understanding and successful problem solving (Gagatsis & Shiakalli, 2004).

This sequence of stages suggests that children may have informal arithmetic abilities that are not apparent when presented with symbolic arithmetic problems and that their abilities in different forms of arithmetic may not emerge at the same time. In support of this notion, research shows that preschoolers are able to solve word problems if allowed to use objects or drawings, although they are unable to do so in the absence of such props (Ginsburg & Russell, 1981; Riley, Greeno, & Heller, 1983). Levine et al. (1992) further examined the development of skills for solving arithmetic problems presented verbally (word problems, and number-fact problems such as "what is four plus two?") and nonsymbolically (object-based operations with calculations identical to the verbal problems) in children between 4 and 6 years of age. The researchers found that children performed better on nonsymbolic problems than on verbal problems across all age groups. Whereas 4-year-olds achieved some success on nonsymbolic problems, comparable levels of performance were not achieved until 5.5–6.5 years old on verbal problems. These findings suggest that children are able to calculate by using physical referents prior to being able to solve simple word problems. In addition, several other studies showed that children can solve simple arithmetic word problems before they comprehend written expressions (Ginsburg, 1977).

The development of different forms of arithmetic is central to our understanding of early arithmetic learning and teaching; however, little attempt has been made to elucidate the cognitive underpinnings of this development during the preschool years. This leaves unclear the very early sources of individual differences in different forms of arithmetic learning. The present study seeks to add to this literature by examining the extent to which two sets of general cognitive skills, specifically visual-spatial and language skills, are related to young children's competencies in three forms of arithmetic that vary in modes of representation. We focus on visual-spatial and language skills because domain-general (Mix, Huttenlocher, & Levine, 2002; Simon, 1997) and domain-specific (Dehaene, 1992; Gelman & Gallistel, 1978) accounts of numerical development both posit that these skills play vital roles in arithmetic learning. For example, Mix et al. (2002) argues that number development can be explained solely in terms of domain-general processes, including visual-spatial (e.g., processing contour length) and language (e.g., naming). In his domain-specific account, Dehaene (1992, 2011) suggests that a space-dominant area (the posterior superior parietal lobule) and a language-dominant area (the left angular gyrus) in the brain, alongside a quantity-dominant area (the intraparietal sulcus), are involved in arithmetic processing. Visual-spatial and language skills are basic cognitive skills that show genetic influences (Plomin, 1999); however, both of them can be taught and substantially improved in home and school settings (Elbro & Petersen, 2004; Uttal et al., 2013). Recent research has demonstrated the importance of these skills for arithmetic learning (e.g., Fuchs et al., 2006; Krajewski & Schneider, 2009; LeFevre et al., 2010; Träff, 2013; Zhang, Koponen, Räsänen, Aunola, Lerkkanen, & Nurmi, 2014).

1.2. *The role of visual-spatial skills in arithmetic learning*

Visual-spatial skills are essential for arithmetical learning. Geary (1993) even argues that one subtype (the visual-spatial subtype) of developmental dyscalculia results from specific deficits in this skill set. Visual-spatial skills are not a unitary construct. Linn and Petersen (1985) distinguish three types, namely spatial perception (identifying spatial relations among task components in spite of distracting information, such as identifying an object whose orientation is different from the others), mental rotation (mentally rotating a 2-D or 3-D object), and spatial visualization (processing complicated, multi-step manipulations, often analytical, of spatial information). Most studies linking these skills to arithmetic focuses on primary and secondary school students (Boonen, van der Schoot, van Wesel, de Vries, & Jolles, 2013; for a meta-analysis, see Friedman, 1992). They generally indicate that visual-spatial skills correlate modestly to successfully completing a wide range of arithmetic tasks, and that this correlation extends to nonsymbolic arithmetic.

Recent research has increasingly documented the relation from visual-spatial skills to early grasp of arithmetic among younger children. Gunderson, Ramirez, Beilock, and Levine (2012) found that children's mental rotation at age five predicted their linear number line knowledge at age six and symbolic arithmetic (measured by a tool combining story problems with Arabic digits) at age eight. Barnes et al. (2011) reported that spatial visualization at age five were associated concurrently with nonsymbolic arithmetic as well as oral counting. More recently, we (Zhang et al. 2014) showed that spatial visualization at age six predicted oral counting at age seven and the growth trajectory of written arithmetic from 7 to 10 years of age. Taken together, these studies suggest that visual-spatial skills are crucial for different forms of arithmetic learning; however, they focus mainly on mental rotation and spatial visualization and largely ignore the role of spatial perception. Indeed, Friedman (1992) even argues that simple tasks of spatial perception, as compared with tasks of mental rotation and spatial visualization, are not considered spatial reasoning (i.e., imagining of spatial transformations) and thus have the least in common with the process of arithmetic problem solving (e.g., operations).

Notably, whereas arithmetic operations may require reasoning and transformations, early understanding of numbers and number relations, which provides a basis for children's later arithmetic learning (Jordan, Kaplan, Ramineni, & Locuniak, 2009), may not necessarily involve these sophisticated visual-spatial skills. For example, recognizing the Arabic code of numbers requires only basic visual functions (Dehaene, 1992). Moreover, the spatial layout of numbers along a mental number line, which plays a crucial role in children's numerical and arithmetic understanding, develop from a logarithmically-compressed form to a linear form (Siegler & Booth, 2004); such development simply involves the spacing of numbers along the number line and likewise may not require spatial reasoning or transformations. Therefore, we speculate that even very basic visual-spatial skills may aid children's mastery of numerical knowledge and, in turn, facilitate their learning of arithmetic. Consequently, the focus of this study is on a basic, understudied visual-spatial function, namely spatial perception.

The ability to perceive spatial relations (e.g., orientation) can be observed in newborns and continues to develop through early childhood (Spelke, 2000). Moreover, the development of spatial perception in early childhood is accompanied by considerable individual variations (Lin et al., 2012). Prior studies suggest that these variations can be captured easily during early childhood: Unlike tests of spatial visualization and mental rotation (e.g., Mental Rotation Test, Vandenberg & Kuse, 1978) where