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First-graders' knowledge of multiplicative reasoning before formal instruction in this domain

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ABSTRACT

Our study investigated children's knowledge of multiplicative reasoning (multiplication and division) at the end of Grade 1, just before the start of formal instruction on multiplicative reasoning in Grade 2. A large sample of children ($N = 1176$) was assessed in a relatively formal test setting, using an online test with 28 multiplicative problems of different types. On average, the children correctly answered more than half (58%) of the problems, including several bare number problems. This indicates that before formal instruction on multiplicative reasoning, children already have a considerable amount of knowledge in this domain, which teachers can build on when teaching them formal multiplication and division. Using analysis of variance and cross-classified multilevel regression analysis, we identified several predictors of children's pre-instructional multiplicative knowledge. With respect to the characteristics of the multiplicative problems, we found that the problems were easiest to solve when they included a picture involving countable objects, and when the multiplicative situation was of the equal groups semantic structure (e.g., 3 boxes of 4 cookies). Regarding student characteristics, pre-instructional multiplicative knowledge was higher for children with higher-educated parents. Finally, the mathematics textbook used in school appeared to have influenced children's pre-instructional multiplicative knowledge.

1. Introduction

Children usually have already built up a considerable amount of mathematical knowledge before they get their first formal instruction in mathematics (e.g., Aubrey, 1994; Carpenter & Moser, 1984; Ginsburg, Klein, & Starkey, 1998). This knowledge is often referred to as informal knowledge (e.g., Baroody, 1987; Ginsburg et al., 1998; Olivier, Murray, & Human, 1990), and is constructed in response to everyday experiences (e.g., Ginsburg et al., 1998; Leinhardt, 1988). Many mathematics educators have stated the importance of building on children's informal mathematical knowledge when teaching them mathematics (e.g., Baroody, 1987; Ginsburg, 1977; Hiebert, 1984; Leinhardt, 1988). They argue that through their informal knowledge children can give meaning to the formal symbols and procedures of mathematics (e.g., Baroody, 1987; Hiebert, 1984). Not building on the knowledge children bring with them may result in children acquiring superficial knowledge without understanding (e.g., Baroody, 1987; Hiebert, 1984), leading, for example, to the erroneous use of mathematical procedures and difficulties in memorizing them (e.g., Baroody, 1987; Olivier et al., 1990).

This building on children's existing knowledge is not only important when children have their first lessons in mathematics, but is also relevant later in the learning process, when a new mathematics domain, such as multiplication, is introduced (e.g., Kouba & Franklin, 1993; Mack, 1995). In this case, children bring with them informal knowledge about multiplication acquired through everyday experiences, as well as prior knowledge acquired from formal mathematics instruction on the related domain of addition. Also, earlier mathematics instruction may have involved preparatory multiplicative activities. As in this case it is hard to distinguish knowledge that is acquired outside school (informal knowledge) from knowledge that is acquired in earlier mathematics lessons, we prefer to speak of pre-instructional knowledge of a certain mathematics domain, including all the knowledge that children have available before formal instruction on that domain starts, regardless of its source.

Despite the stated importance of connecting the formal mathematics to children's (informal) pre-instructional knowledge, researchers have found that teachers often fail to make these connections (e.g., Aubrey, 1994; Leinhardt, 1988). A possible explanation for this may be that teachers underestimate children's pre-instructional knowledge. Several studies have found evidence

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for such underestimations of children's mathematics abilities (e.g., Grassmann, Mirwald, Klunter, & Veith, 1995; Lee & Ginsburg, 2009; Selter, 1993; Van den Heuvel-Panhuizen, 1996). For teachers to be able to build on children's prior knowledge, it is at least necessary that they are aware of and acknowledge this knowledge. In fact, it is argued that knowledge of what children already know about a particular mathematics domain should be an important aspect of teachers' didactical knowledge (Carpenter, Fennema, & Franke, 1996). Therefore, it is crucial that the pre-instructional knowledge of children is revealed.

This paper describes a study into children's pre-instructional knowledge in the domain of multiplicative reasoning.[1] We aimed to map children's understanding of multiplication and division just before they start receiving formal instruction on this domain.

2. Theoretical background and research questions

2.1. Multiplicative reasoning

The mathematics domain of multiplicative reasoning, comprising multiplication and division, is clearly distinguished from the domain of additive reasoning, including addition and subtraction (e.g., Clark & Kamii, 1996; Schwartz, 1988; Vergnaud, 1983). In contrast to additive reasoning, in which quantities of the same type are added or subtracted (e.g., 2 *cookies* and 3 *cookies* are 5 *cookies* altogether), multiplicative reasoning involves quantities of different types (e.g., 3 *boxes* with 4 *cookies per box* means 12 *cookies* altogether). Accordingly, Schwartz (1988, p. 41) asserted that addition and subtraction are "referent preserving compositions", whereas multiplication and division are "referent transforming compositions". A multiplicative situation is characterized by a group structure which involves sets (groups, e.g., boxes) of items with in each set the same number of items (e.g., cookies) (see Greer, 1992). This distinction between items and sets of items was emphasized by Nantais and Herscovics (1990, p. 289), stating that "a situation is perceived as being multiplicative when the whole is viewed as resulting from the repeated iteration of a one-to-one or a one-to-many correspondence". In this definition, a one-to-one correspondence refers to the situation where there is one item in each set, whereas in the case of a one-to-many correspondence, the sets contain more than one item. Although multiplication problems can be calculated by repeated addition or counting in groups, which is how they are often introduced to children, multiplication is conceptually different from addition, since one of the operands denotes the number of times a value should be added (the number of sets), instead of a value to be added (see, e.g., Clark & Kamii, 1996).

Multiplicative reasoning has an important place in primary mathematics learning, since it is required as a foundation for the understanding of more complex mathematical concepts in the multiplicative conceptual field (Vergnaud, 1983), such as ratio, fractions, and linear functions. These concepts are all related to proportional reasoning, which Lesh, Post, and Behr (1988, p. 94) described as both the "capstone" of primary school mathematics and the "cornerstone" of the mathematics that follows. Besides its importance for later mathematical understanding, multiplicative reasoning is implicitly necessary for understanding place value (e.g., interpreting 63 as 6 tens and 3 ones; see Nunes et al., 2009).

Formal instruction on multiplicative reasoning generally starts in the second grade (e.g., in the Netherlands; see Van den Heuvel-Panhuizen, 2008) or third grade (e.g., in the US; see NCTM, 2006) of primary school, after addition and subtraction have been taught. Often, division is formally introduced after multiplication (see Mulligan & Mitchelmore, 1997; Van den Heuvel-Panhuizen, 2008).

2.2. Previous research on children's pre-instructional knowledge of multiplicative reasoning

Earlier studies have revealed that young children already have some understanding of multiplicative relations before the domain is formally introduced in school (e.g., Anghileri, 1989; Kouba, 1989; Mulligan & Mitchelmore, 1997; Nunes & Bryant, 1996; see also Ter Heege, 1985). In Anghileri's (1989) study, for example, first-grade students could solve an average of 56% of physically presented multiplication tasks, and in Kouba's (1989) study, first graders could already solve some simple multiplication and division word problems (25% correct on average). Furthermore, in a longitudinal study by Mulligan and Mitchelmore (1997), Australian children at the beginning of Grade 2 correctly solved an average of 31% of multiplicative word problems, increasing to 48% at the end of Grade 2 and 55% at the beginning of Grade 3 (all these measurements were before formal instruction on multiplicative reasoning). Carpenter and colleagues found that even many kindergartners were able to solve a variety of multiplication and division word problems (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993).

In all previous studies on children's pre-instructional knowledge of multiplicative reasoning, the problems were either presented in a physical context (e.g., Anghileri, 1989) or the children were allowed and encouraged to use physical materials, such as counters and blocks, to construct a physical representation for themselves (e.g., Kouba, 1989; Mulligan & Mitchelmore, 1997). The majority of the children did actually employ these materials (Carpenter et al., 1993; Kouba, 1989). This probably helped them in modeling the problem situation and in keeping track of counting and repeated addition or subtraction activities, and thus made it easier to solve the problems (see, e.g., Ibarra & Lindvall, 1982; Levine, Jordan, & Huttenlocher, 1992). From the previous studies, then, it is not known whether children also show this knowledge when no physical representation is offered or can be created by the child. Moreover, the studies have only focused on problems presented in a context and not on bare number problems, like "$2 \times 4 =$ ___" or "2 times 4 is ___".[2] Furthermore, in the aforementioned studies the children were assessed in individual interviews, in which the interviewer could have encouraged the children in reaching a solution. It has indeed been found that one-to-one interview settings may help students in solving mathematics problems (Caygill & Eley, 2001). In the previous studies it was not investigated whether children also show pre-instructional knowledge of multiplicative reasoning when they are assessed in a more formal setting, in which there is no interviewer sitting next to them. Finally, the previous studies were small-scale studies, which may make results hard to generalize.

2.3. Possible factors influencing children's pre-instructional multiplicative knowledge

Research suggests that there are several factors that may influence the pre-instructional multiplicative knowledge children display. Below we discuss the most important characteristics that we found in the literature. First of all, the characteristics of the problems offered to the children may affect their performance. In addition, children's gender, the educational level of their parents,

[1] We use the terms *pre-instructional knowledge of multiplicative reasoning* and *pre-instructional multiplicative knowledge* interchangeably.

[2] Baroody (1999) did study first-graders' abilities in solving bare number multiplication problems. However, in his study the children were first introduced to the × symbol, which can be considered a first formal instruction on multiplication. Baroody's study, thus, was not performed before formal instruction.